

Money and Public Finance

Budget Accounting:

$$G_t + i_{t-1} B_{t-1}^T = T_t + (B_t^T - B_{t-1}^T) + RCB_t$$

$$(B_t^M - B_{t-1}^M) + RCB_t = i_{t-1} B_{t-1}^M + (H_t - H_{t-1})$$

$$G_t + i_{t-1} B_{t-1}^T + B_t^M - B_{t-1}^M + RCB_t = T_t + (B_t^T - B_{t-1}^T)$$

$$G_t + i_{t-1} (B_{t-1}^T - B_{t-1}^M) = T_t + [(B_t^T - B_t^M) - (B_{t-1}^T - B_{t-1}^M)] + (H_t - H_{t-1})$$

$$\frac{G_t}{P_t} + i_{t-1} \frac{B_{t-1}}{P_t} = \frac{T_t}{P_t} + \frac{(B_t - B_{t-1})}{P_t} + \frac{H_t - H_{t-1}}{P_t}$$

$$x_t = \frac{x_t}{P_t} \quad \frac{B_{t-1}}{P_t} = \frac{B_{t-1}}{P_{t-1}} \cdot \frac{P_{t-1}}{P_t} = \frac{b_{t-1}}{1+\pi_t}$$

$$\pi_t = \frac{P_t - P_{t-1}}{P_{t-1}} = \frac{P_t}{P_{t-1}} - 1 \Rightarrow \frac{P_{t-1}}{P_t} = \frac{1}{1+\pi_t}$$

$$g_t + i_{t-1} \frac{b_{t-1}}{1+\pi_t} = t_t + b_t - \frac{b_{t-1}}{1+\pi_t} + h_t - \frac{h_{t-1}}{1+\pi_t}$$

$$\bar{r}_{t-1} = \frac{1+i_{t-1}}{1+\pi_t} - 1$$

$$g_t + \bar{r}_{t-1} b_{t-1} = t_t + b_t - b_{t-1} + h_t - \frac{h_{t-1}}{1+\pi_t}$$

$$1+i_{t-1} = (1+\bar{r}_{t-1})(1+\pi_t^e)$$

$$r_{t-1} - \bar{r}_{t-1} = \frac{(\pi_t - \pi_t^e)(1+\bar{r}_{t-1})}{1+\pi_t}$$

$$g_t + r_{t-1} b_{t-1} - (r_{t-1} - \bar{r}_{t-1}) b_{t-1} = t_t + b_t - b_{t-1} + h_t - \frac{h_{t-1}}{1+\pi_t}$$

$$g_t + r_{t-1} b_{t-1} = t_t + (b_t - b_{t-1}) + \frac{(\pi_t - \pi_t^e)(1+\bar{r}_{t-1})}{1+\pi_t} b_{t-1} + \left[h_t - \frac{1}{1+\pi_t} h_{t-1} \right]$$

عواقب الفرب

$$s_t = \frac{H_t - H_{t+1}}{P_t} = h_t - h_{t-1} + \left(\frac{\pi_t}{1+\pi_t} \right) h_{t-1}$$

$$\theta \frac{\pi}{1+\pi} h = \frac{\theta}{1+\theta} h$$

سؤال:

مقدار التغير في الدين العام

$$d_t = b_t + h_t$$

$$g_t + r_{t-1} d_{t-1} = t_t + (d_t - d_{t-1}) + \frac{(\pi_t - \pi_t^e)(1+\bar{r}_{t-1})}{1+\pi_t} d_{t-1} + \frac{i_{t-1}}{1+\pi_t} h_{t-1}$$

$$\bar{s} = \frac{i}{1+\pi}$$

$$\{g_{t+i}, b_{t+i}\}_{t=\infty} = 0$$

Intertemporal Budget Balance

$$\{g_{t+i}, b_{t+i}\}_{i=0}^{\infty} = 0$$

Intertemporal Budget Balance

$$g_t + r_{t-1} b_{t-1} = t_t + (b_t - b_{t-1}) + s_t$$

$$y_t = \alpha + \beta y_{t-1} + \varepsilon_t$$

$$x_t = \theta + \gamma x_{t-1} + u_t$$

$$y_t = \alpha \sum_{i=0}^{\infty} \beta^i + \beta^{t+1} y_{-1} + \sum_{i=0}^{\infty} \beta^i \varepsilon_{t-i}$$

$$(1+r) b_{t-1} + \sum_{i=0}^{\infty} \frac{g_{t+i}}{(1+r)^i} = \sum_{i=0}^{\infty} \frac{t_{t+i}}{(1+r)^i} + \sum_{i=0}^{\infty} \frac{s_{t+i}}{(1+r)^i} + \lim_{i \rightarrow \infty} \frac{b_{t+i}}{(1+r)^i}$$

$$(1+r) b_{t-1} = - \sum_{i=0}^{\infty} \frac{\Delta_{t+i}}{(1+r)^i}$$

$$\Delta = g - t - s$$

Money and Fiscal Policy Framework:

$$W = B + M$$

Deficits and Inflation:

$$R = 1+r$$

$$b_{t-1} = -R^{-1} \sum_{i=0}^{\infty} R^{-i} (g_{t+i} - t_{t+i} - s_{t+i})$$

$$s_t^f = t - g$$

$$b_{t-1} = R^{-1} \sum_{i=0}^{\infty} R^{-i} s_{t+i}^f + R^{-1} \sum_{i=0}^{\infty} R^{-i} s_{t+i}$$

Unpleasant monetarist arithmetic

Ricardian and Non-Ricardian Fiscal Policies:

$$\frac{M}{P} \uparrow \quad \text{Metzler (1951)}$$

$$g = 0$$

$$(1+r_{t-1}) b_{t-1} = t_t + b_t + s_t$$

$$c_t + M_t + b_t = y_t + (1+r_{t-1}) b_{t-1} + \frac{M_t - M_{t-1}}{1+r_t} (-t_t) *$$

$$1+r_{t-1} = \frac{1+r_t}{1+r_t}$$

Aiyagari and Gertler (1985)

منه ψ (نسبة الدين إلى الناتج المحلي الإجمالي)

$$0 \leq \psi \leq 1 \quad \text{Ricardian} \Rightarrow \psi = 1$$

if $\psi < 1$ non-Ricardian

$$T_t = \psi (1+r_{t-1}) b_{t-1} \Rightarrow T_{t+1} = \psi (1+r_t) b_t$$

منه ψ (نسبة الدين إلى الناتج المحلي الإجمالي)

$$T_t = t_t + E(T_{t+1}) = t_t + E[\psi (1+r_t) b_t]$$

از شرط تعادل بود

$$T_t = t_t + E\left(\frac{T_{t+1}}{1+r_t}\right) = t_t + E_t\left[\frac{\psi(1+r_t)b_t}{1+r_t}\right]$$

$$T_t = t_t + \psi b_t$$

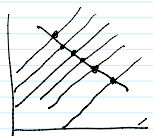
$$T_t = \psi(1+r_{t-1})b_{t-1} = t_t + \psi b_t$$

$$\star \rightarrow t_t = \psi(R_{t-1}b_{t-1} - b_t) \Rightarrow R = 1+r$$

$$c_t + m_t + (1-\psi)b_t = y_t + (1-\psi)R_{t-1}b_{t-1} + \frac{m_{t+1}}{1+r_t}$$

if $\psi < 1$ $w = m + (1-\psi)b$

$$c_t + w_t + \frac{i_t + 1 - m_{t+1}}{1+r_t} = y_t + R_{t-1}w_{t-1}$$



$$U = \ln C_t + \delta \ln M_t$$

$$\frac{U'_M}{U'_C} = \frac{i_t}{1+i_t}$$

$$\frac{\delta}{\frac{1}{C_t}} = \frac{i_t}{1+i_t}$$

$$m_t = \frac{\delta C_t (1+i_t)}{i_t}$$

$$C_t \delta \frac{1+i_t}{i_t}$$

$$C_{t+1} = \beta(1+r_t)C_t$$

$$y + R_{t-1}w_{t-1} = c_t + w_t + \left(\frac{i_t - 1}{1+r_t}\right) \delta \left(\frac{1+i_{t-1}}{i_{t-1}}\right) \frac{c_t}{\beta(1+r_{t-1})}$$

$$= \left(1 + \frac{\delta}{\beta}\right) c_t + w_t$$

$$g = 0 \quad I = 0 \quad y = c$$

$$R_{t-1}w_{t-1} = \frac{\delta}{\beta} y + w_t$$

$$w_{t-1} = w_t = w^{SS} = \frac{\delta y}{\beta(R-1)}$$

$$w = \frac{M + (1-\psi)\beta}{P} = \frac{\delta y}{\beta(R-1)}$$

$$P^{SS} = \left(\frac{\beta r^{SS}}{\delta y}\right) [M + (1-\psi)\beta]$$

Fiscal Theory of the Price Level

FTPL

$$\lambda = \frac{M}{M+\beta}$$

$$P^{SS} = \left(\frac{\beta r^{SS}}{\delta y}\right) [1 - \psi(1-\lambda)] (M+\beta)$$

$$p^{SS} = \left(\frac{\beta y^{SS}}{\delta y} \right) [1 - \psi(1-\lambda)] (M + B)$$

Sargent and Wallace
Perfect foresight

$$\frac{u'_m(c_t, m_t)}{u'_c(c_t, m_t)} = \frac{i_t}{1+i_t}$$

$$u(c_t, m_t) = \frac{[a c_t^{1-b} + (1-a) m_t^{1-b}]^{1-\phi}}{1-\phi}$$

$$m_t = \frac{M_t}{P_t} = \left[\frac{i_t}{1+i_t} \left(\frac{a}{1-a} \right) \right]^{-\frac{1}{b}} c_t$$

$$R_m = 1+i \Rightarrow \frac{i_t}{1+i_t} = \frac{R_{m,t} - 1}{R_{m,t}}$$

$$\boxed{\frac{M_t}{P_t} = f(R_{m,t})}$$

$$f(R_{m,t}) = \left[\frac{R_{m,t} - 1}{R_{m,t}} \left(\frac{a}{1-a} \right) \right]^{-\frac{1}{b}} c_t$$

$$\frac{M_0}{P_0} = f(R_m)$$

$$P_0 = \frac{M_0}{f(R_m)}$$

$$g + r b_{t-1} = t + (b_t - b_{t-1}) + m_t - \left(\frac{1}{1+r_t} \right) m_{t-1}$$

$g, b, t \rightarrow c$

$$g + r b = t + m - \frac{1}{1+r} m$$

$$\left(1+r = \frac{1}{\beta} \right) \Rightarrow r = \left(\frac{1}{\beta} - 1 \right)$$

$$g + \left(\frac{1}{\beta} - 1 \right) b = t + \left(\frac{r}{1+r} \right) m$$

$$1+i = \frac{1+r}{1+\pi} = R_m \Rightarrow R_m = \frac{1+\pi}{\beta}$$

$$\pi = R_m \beta - 1$$

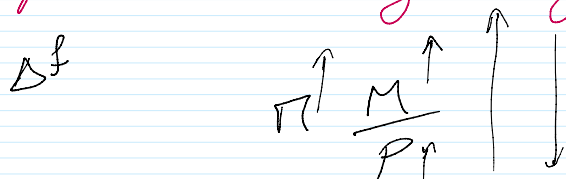
$$g + \left(\frac{1}{\beta} - 1 \right) b = t + \left(\frac{\beta R_m - 1}{\beta R_m} \right) f(R_m)$$

Fiscal Dominance

Active Fiscal Policy

Passive Monetary Policy

Equilibrium Seigniorage:



Cagan (1956)

$$\frac{m_t - m_{t+1}}{P_t}$$

$$\sum_{t=0}^{\infty} \beta^t u(c_t, m_t)$$

$$c_t + b_t + m_t = y_t - \tau_t + (1+r) b_{t-1} + \frac{m_{t-1}}{\pi_t}$$

$$\pi_t = 1 + \pi_t = \frac{P_t}{P_{t-1}} \quad \text{Cagan's}$$

$$w_t = b_t + m_t, \quad R_t = 1 + r_t$$

$$\begin{aligned} c_t + w_t &= y_t - \tau_t + R_{t-1} w_{t-1} - \left(\frac{R_{t-1} \pi_t - 1}{\pi_t} \right) m_{t-1} \\ &= y_t - \tau_t + R_{t-1} w_{t-1} - \frac{i_{t-1}}{\pi_t} m_{t-1} \end{aligned}$$

$$\frac{u'_m(c_t, m_t)}{u'_c(c_t, m_t)} = \frac{i_t}{1 + i_t}$$

$$u'_c(c_t, m_t) = \frac{\beta i_t}{\pi_{t+1}} u'_c(c_{t+1}, m_{t+1})$$

$$u'_m(c_t, m_t) = \frac{i_t}{R_t \pi_{t+1}} u'_c(c_t, m_t) = \frac{i_t}{1 + i_t} u'_c(c_t, m_t)$$

$$R_t = (1+r_t) = \frac{1+i_t}{1+\pi_{t+1}} = \frac{1+i_t}{\prod_{t+1}}$$

$$u(c_t, m_t) = \ln c_t + m_t (B - D \ln m_t)$$

اینجا از فرمول ربح، یعنی $\frac{1+i_t}{1+\pi_{t+1}}$ استفاده می‌کنیم.
 در اینجا $\frac{1+i_t}{1+\pi_{t+1}}$ را به جای R_t قرار می‌دهیم.

$$u'_c = \frac{1}{c_t}$$

$$u'_m = B - D \ln m_t - D \frac{m_t}{m_t} = B - D - D \ln m_t$$

$$\frac{u'_m}{u'_c} = \frac{B - D - D \ln m_t}{\frac{1}{c_t}} = \frac{i_t}{1+i_t}$$

$$B - D - D \ln m_t = \frac{i_t}{1+i_t} \cdot \frac{1}{c_t}$$

$$D \ln m_t = B - D - \frac{\omega_t}{c_t}$$

$$\ln m_t = \left(\frac{B}{D} - 1\right) - \frac{\omega_t}{D c_t}$$

$$m_t = e^{\frac{B}{D} - 1} \cdot e^{-\frac{\omega_t}{D c_t}}$$

$$m_t = A e^{-\frac{\omega_t}{D c_t}}$$

$$m = k e^{-\alpha \pi e}$$

$$\frac{i m}{1+\pi} = \frac{i}{1+i} \cdot m = \frac{(1+r)m}{1+i}$$

$$\omega_t = \frac{i_t}{1+i}$$

$$\bar{s} = (1+r) \frac{i}{1+i} m = (1+r) \frac{i}{1+i} A \exp\left(-\frac{i}{D(1+i)}\right)$$

$$\frac{\partial \bar{s}}{\partial \pi} = \frac{\partial \bar{s}}{\partial \omega} \cdot \frac{\partial \omega}{\partial i} \cdot \frac{\partial i}{\partial \pi}$$

$$\omega = \frac{i}{1+i} \quad \frac{\partial \omega}{\partial i} = \frac{1}{(1+i)^2}$$

$$\omega = \frac{i}{1+i} \quad \frac{\partial \omega}{\partial i} = \frac{1}{(1+i)^2}$$

$$(1+r) = \frac{1+i}{1+\pi} \Rightarrow i = (1+r)(1+\pi) - 1$$

$$\frac{\partial i}{\partial \pi} = (1+r)$$

$$\frac{\partial \bar{S}}{\partial \pi} = \frac{\partial \bar{S}}{\partial \omega} \cdot \frac{\partial \omega}{\partial i} \cdot \frac{\partial i}{\partial \pi}$$

$$\bar{S} = (1+r)A\omega \exp\left(-\frac{\omega}{DC}\right)$$

$$\begin{aligned} \frac{\partial \bar{S}}{\partial \omega} &= (1+r)A \exp\left(-\frac{\omega}{DC}\right) - (1+r) \frac{A\omega}{DC} \exp\left(-\frac{\omega}{DC}\right) \\ &= (1+r)A \left[1 - \frac{\omega}{DC}\right] \exp\left(-\frac{\omega}{DC}\right) = \frac{\bar{S}}{\omega} \left[1 - \frac{\omega}{DC}\right] \end{aligned}$$

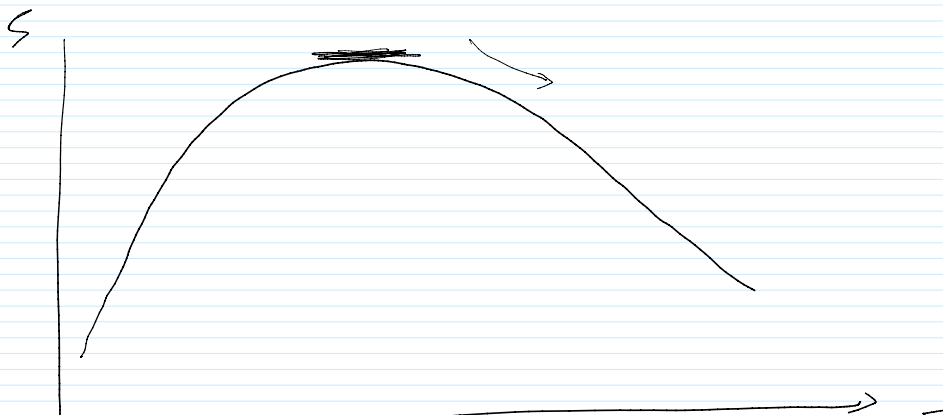
$$\bar{S} = (1+r)A\omega \exp\left(-\frac{\omega}{DC}\right)$$

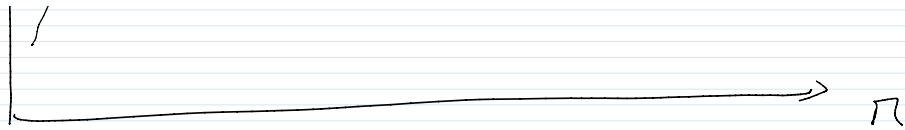
$$\frac{\partial \bar{S}}{\partial \omega} = 0 \Rightarrow 1 - \frac{\omega}{DC} = 0 \quad \omega = DC$$

$$\frac{i}{1+i} = DC$$

$$\pi^{\max} = \left(\frac{1}{1+r}\right) \frac{1}{(1-DC)} - 1$$

$$\frac{1+i}{1+\pi} = 1+r$$





Cagan's Model:

$$\Delta f = \frac{\dot{h}}{h} \frac{H}{PY} = \theta h$$

$$h = \exp(-\alpha \pi^e)$$

$$\Delta f = \theta e^{-\alpha \pi^e}$$

$$\pi = \theta - \mu \quad \rightarrow \text{Cagan's Model}$$

$$\pi^e = \pi$$

$$\pi_t = \pi_{t-1} = E_t \pi_{t+1} = \bar{\pi}$$

$$\Delta f = \theta e^{-\alpha(\theta - \mu)}$$

$$\Delta f = 0 \Rightarrow \theta = 0$$

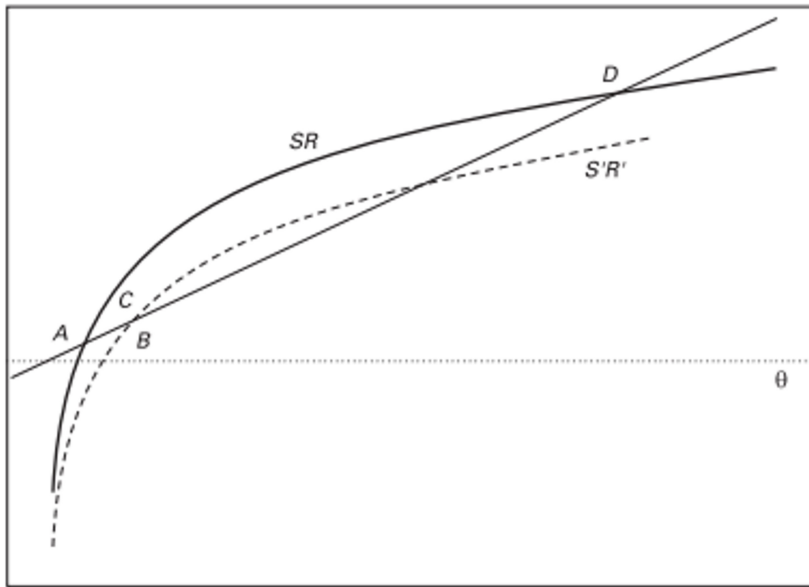
$$\frac{\partial \Delta f}{\partial \theta} = e^{-\alpha(\theta - \mu)} - \alpha \theta e^{-\alpha(\theta - \mu)} = 0$$

$$\theta = \frac{1}{\alpha}$$

$$\pi = \theta - \mu = \frac{1}{\alpha} - \mu$$

$$\Delta^* = \frac{1}{\alpha} e^{-\alpha(\frac{1}{\alpha} - \mu)} = \frac{1}{\alpha} e^{\alpha\mu - 1}$$

Inflation



$$\frac{\partial \pi^e}{\partial t} = \dot{\pi}^e = \eta (\pi - \pi^e)$$

$$h = e^{-\alpha \pi^e}$$

$$\frac{\dot{h}}{h} = \theta - \mu - \pi = -\alpha \dot{\pi}^e = -\alpha \eta (\pi - \pi^e)$$

$$\pi = \frac{\theta - \mu - \alpha \eta \pi^e}{1 - \alpha \eta}$$

$$\dot{\pi}^e = \frac{\eta (\theta - \mu - \pi^e)}{1 - \alpha \eta}$$

$$\alpha \eta < 1 \Rightarrow \eta < \frac{1}{\alpha}$$

Rational Hyperinflation

$$m_t - p_t = -\alpha (E_t p_{t+1} - p_t)$$

$$(1 + \alpha) p_t = m_t + \alpha E_t p_{t+1}$$

$$p_t = \frac{1}{1 + \alpha} m_t + \frac{\alpha}{1 + \alpha} E_t p_{t+1}$$

Stiglitz

$$m_t = \theta_0 + (1-\gamma)\theta_1 t + \gamma m_{t-1}$$

عرض جزئي

$$m_{t-1} = \theta_0 + (1-\gamma)\theta_1(t-1) + \gamma m_{t-2}$$

سحب جزئي

$$m_t - m_{t-1} = (1-\gamma)\theta_1 + \gamma(m_{t-1} - m_{t-2})$$

$$mg_t = (1-\gamma)\theta_1 + \gamma mg_{t-1}$$

$$\bar{mg} = (1-\gamma)\theta_1 + \gamma \bar{mg}$$

$$\boxed{\bar{mg} = \theta_1}$$

$$P_t = \frac{1}{1+\alpha} m_t + \frac{\alpha}{1+\alpha} E_t P_{t+1}$$

$$E_t P_{t+1} = \frac{1}{1+\alpha} E_t m_{t+1} + \frac{\alpha}{1+\alpha} E_t E_{t+1} P_{t+2}$$

$E_t P_{t+2}$

$$m_t = \theta_0 + (1-\gamma)\theta_1 t + \gamma m_{t-1}$$

$$E_t m_{t+1} = \theta_0 + (1-\gamma)\theta_1(t+1) + \gamma m_t$$

$$P_t = \frac{\alpha[\theta_0 + (1-\gamma)\theta_1(1+\alpha)]}{1+\alpha(1-\gamma)} + \frac{\alpha(1-\gamma)\theta_1}{1+\alpha(1-\gamma)} t + \frac{1}{1+\alpha(1-\gamma)}$$

$$P_t = A_0 + A_1 t + A_2 m_t$$

$$E_t P_{t+1} = A_0 + A_1(t+1) + A_2 E_t m_{t+1} = A_0 + A_1(t+1) + A_2[\theta_0 + (1-\gamma)\theta_1(t+1) + \gamma m_t]$$

سحب جزئي $\rightarrow \theta_1$

$$P_t = A_0 + A_1 t + A_2 m_t + B_t = \frac{m_t}{1+\alpha} + \frac{\alpha[A_0 + A_1(t+1) + A_2 E_t m_{t+1}]}{1+\alpha}$$

$$A_0 = \frac{\alpha[\theta_0 + (1-\gamma)\theta_1(1+\alpha)]}{1+\alpha(1-\gamma)}, \quad A_1 = \frac{\alpha(1-\gamma)\theta_1}{1+\alpha(1-\gamma)}, \quad A_2 = \frac{1}{1+\alpha}$$

m_t

$\theta_{t+1} + \gamma m_t$

$t+1 + E_t B_{t+1}$

$(1-\gamma)$

$$B_t = \frac{\alpha}{1+\alpha} E_t B_{t+1}$$

$$B_{t+1} = k B_t, \quad k = \frac{1+\alpha}{\alpha} > 1$$

$$k-1 = \frac{1}{\alpha}$$

The Fiscal Theory of the Price Level:

$$D_t + P_t y_t - T_t \geq P_t c_t + M_t^d + B_t^d = P_t c_{t+1} \left(\frac{1+i_t}{1+r_t} \right) M_{t+1}^d + \left(\frac{1}{1+r_t} \right) D_{t+1}^d$$

لدينا

$$D_{t+1}^d = (1+i_t) B_t^d + M_t^d$$

$$d_t + y_t - \tau_t \geq c_t + m_t^d + b_t^d = c_t + \left(\frac{1+i_t}{1+r_t} \right) m_{t+1}^d + \left(\frac{1}{1+r_t} \right) d_{t+1}^d$$

$$1+r_t = (1+i_t)(1+r_{t+1})$$

$$\lambda_{t,t+i} = \prod_{j=1}^i \left(\frac{1}{1+r_{t+j}} \right), \quad \lambda_{t,t} = 1$$

$$d_t + \sum_{i=0}^{\infty} \lambda_{t,t+i} (y_{t+i} - \tau_{t+i}) = \sum_{i=0}^{\infty} \lambda_{t,t+i} \left[c_{t+i} + \left(\frac{1+i_{t+i}}{1+r_{t+i}} \right) d_{t+i}^d \right]$$

لدينا

$$P_t g_t + (1+i_{t-1}) B_{t-1} = T_t + M_t - M_{t-1} + B_t$$

$$g_t + d_t = \tau_t + \left(\frac{1+i_t}{1+r_t} \right) m_{t+1} + \left(\frac{1}{1+r_t} \right) d_{t+1}$$

$$d_t + \sum_{i=0}^{\infty} \lambda_{t,t+i} \left[g_{t+i} - \tau_{t+i} - \bar{s}_{t+i} \right] = \lim_{T \rightarrow \infty} \lambda_{t,t+T} d_T = 0$$

$\int_{\mathbb{Z}+i}^d$

o

$$a_t + \sum_{i=0}^{\infty} \frac{1}{(1+i)^{t+i}} (g_{t+i} - \tau_{t+i}) = \frac{K_t}{T} \rightarrow \infty$$

$$\bar{S}_t = \frac{r_t + m_t}{1+i_t}$$

عوض کردن

$$\lim_{T \rightarrow \infty} \int_{t, t+T} dT = 0 \quad P_t \text{ (مجموعه مسارات)}$$

مسیر (یا مسارات) $(g_{t+i}, \tau_{t+i}, S_{t+i}, d_{t+i})_{i \geq 0}$

if $\lim_{T \rightarrow \infty} \int_{t, t+T} dT = 0$ for all paths of $P_{t+i}, i \geq 0$

$\lim_{T \rightarrow \infty} \int_{t, t+T} dT \neq 0$

$$y_t = c_t + g_t \Rightarrow c_t = y_t - g_t$$

$$m_t^d = m_t$$

$$d_t + \sum_{i=0}^{\infty} \frac{1}{(1+i)^{t+i}} \left[g_{t+i} - \tau_{t+i} - \left(\frac{i_{t+i}}{1+i_{t+i}} \right) m_t \right]$$

Ricardian

non-Ricardian

\Rightarrow Ricardian

\Rightarrow Non-Ricardian

$$\left. \begin{array}{l} t+i \end{array} \right] = 0$$

$$d_t = \frac{D_t}{P_t} = \sum_{i=0}^{\infty} \lambda_{t,t+i} \left[\tau_{t+i} + \bar{S}_{t+i} - g_{t+i} \right]$$

$$\frac{M_t}{P_t} = f(1+i)$$

$$i_{t+i} = \bar{i}$$

$$\bar{S}_t = \left(\frac{i_t}{1+i_t} \right) M^d \Rightarrow \bar{S} = \frac{\bar{i}}{1+\bar{i}} f(1+\bar{i})$$

$$P_t^* = \frac{D_t}{\sum_{i=0}^{\infty} \lambda_{t,t+i} \left[\tau_{t+i} + \bar{S}_{t+i} - g_{t+i} \right]}$$

$$M_t = P_t^* f(1+\bar{i})$$

$$\sum_{i=0}^{\infty} \lambda_{t,t+i} \bar{S}_{t+i}$$

Optimal Taxation and Seigniorage:

A partial Equilibrium model:

$t + i$

\bar{g} \bar{R}

$$b_t = R b_{t-1} + g - \tau_t - s_t$$

$$s_t = \frac{M_t - M_{t-1}}{P_t} = m_t - \frac{m_{t-1}}{1+R_t}$$

$$E_t \sum_{i=0}^{\infty} R^{-i} (\tau_{t+i} + s_{t+i}) = R b_{t-1} + \left(\frac{R}{1+R} \right) g$$

$$E_t \lim_{i \rightarrow \infty} R^{-i} b_{t+i} = 0$$

