Session03

Tuesday, October 24, 2023 9:20 AM

1, 3, 4 $\Rightarrow$

معادله اویلر

$$E_t \frac{1+i_t}{1+\pi_{t+1}} = \left[f'\left(\frac{k_t}{1+n}\right) + (1-\delta) \right]$$

معادله فیشر

$$Y_t = r_t K_t + w_t N_t$$

$$f(x,y) = x f'_x + y f'_y$$

$$F(K,N) = K f'_K + N f'_N$$

$$r = f'_K \quad , \quad w = f'_N$$

$$\pi_t = P_t - P_{t-1}$$

$$r_t = f'\left(\frac{k_t}{1+n}\right) - \delta$$

$$E_t \frac{1+i_t}{1+\pi_{t+1}} = 1 + r_t$$

$$1 + i_t = 1 + r_t E_t \pi_{t+1} + r_t + E_t \pi_{t+1}$$

$$r_t = i_t - E_t \pi_{t+1} + r_t E_t \pi_{t+1}$$

$$r_t = i_t - \pi_t$$

Steady State Equilibrium:

First order Difference Equation (AR(1))

$$y_t = \alpha + \beta y_{t-1} + \varepsilon_t \quad , \quad \varepsilon_t \sim i.i.d.\ N(0,1)$$

$$E_t y_t = \alpha + \beta E_t y_{t-1}$$

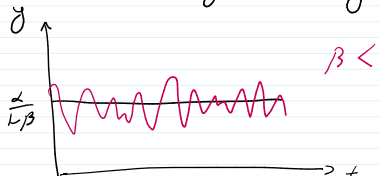
$$\bar{y} = \alpha + \beta \bar{y}$$

$$(1-\beta)\bar{y} = \alpha$$

$$\bar{y} = \frac{\alpha}{1-\beta}$$

$$y_t = \alpha(1+\beta+\beta^2+\dots+\beta^t) + \beta^{t+1} y_{-1} + \varepsilon_t + \beta\varepsilon_{t-1} + \beta^2\varepsilon_{t-2} + \dots + \beta^t \varepsilon_0$$

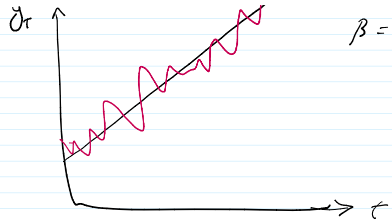
mean-reverting property



$\beta < 1$

$\beta = 1$

$y_t = \alpha + y_{t-1} + \varepsilon_t$



$\beta = 1$

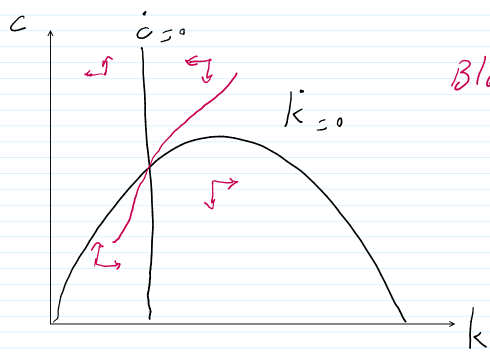
$$y_t = \alpha + y_{t-1} + \varepsilon_t$$

$$y_t = \alpha + y_{t-1} + \varepsilon_t$$

$$\Delta y_t = y_t - y_{t-1} = \alpha + \varepsilon_t$$

Steady State

$$y_t = y_{t-1} = E_t\, y_{t+1} = y^{ss}$$



Blanchard-kahn

$$u'_c(c_t, m_t) = \beta E_t \left[f'_k(k_t) + 1 - \delta \right] u'_c(c_{t+1}, m_{t+1})$$

$$E_t \frac{1 + i_t}{1 + \pi_{t+1}} = f'(k_t) + 1 - \delta$$

$$\frac{u'_m(c_t, m_t)}{u'_c(c_t, m_t)} = \frac{i_t}{1 + i_t}$$

$$u'_m(c_t, m_t) - \beta \left[f_k(k_t) + 1 - \delta \right] u'_c(c_t, m_t) + \beta \frac{E_t u'_c(c_t, m_t)}{1 + \pi_{t+1}}$$

$$f(k_t) + \tau_t + (1-\delta) k_{t-1} + \frac{m_{t-1}}{1 + \pi_t} = c_t + k_t + m_t \quad \Leftarrow$$

$$\cancel{u'_c(c^{ss}, m^{ss})} = \beta \left[f'_k(k^{ss}) + 1 - \delta \right] \cdot \cancel{u'_c(c^{ss}, m^{ss})}$$

$$\Rightarrow \frac{1 + i^{ss}}{1 + \pi^{ss}} = f'(k^{ss}) + 1 - \delta$$

$$\tau + (1 + i_{t-1}) \frac{b_{t-1}}{1 + \pi_t} + \frac{m_{t-1}}{1 + \pi_t} = m_t + b_t$$

$$\tau + (1 + i^{ss}) \frac{b^{ss}}{1 + \pi^{ss}} + \frac{m^{ss}}{1 + \pi^{ss}} = m^{ss} + b^{ss}$$

$$b^{ss} = 0$$

$$\tau = m^{ss}\left(1 - \frac{1}{1+\pi^{ss}}\right)$$

$$n = 0 \qquad \pi^{ss} = \theta$$

$$1 = \beta\left[f_k'(k^{ss}) + (1-\delta)\right]$$

$$f_k'(k^{ss}) = \frac{1}{\beta} - 1 + \delta \qquad\qquad f(k) = k^{\alpha}$$

$$f'(k) = \alpha k^{\alpha-1}$$

$$\alpha {k^{ss}}^{\alpha-1} = \frac{1+(\delta-1)\beta}{\beta}$$

$$k^{ss} = \left[\frac{\alpha\beta}{1+\beta(\delta-1)}\right]^{\frac{1}{1-\alpha}}$$

$$y^{ss} = {k^{ss}}^{\alpha} = \left[\frac{\alpha\beta}{1+\beta(\delta-1)}\right]^{\frac{\alpha}{1-\alpha}}$$

Money Neutrality خنثی پول

$$\tau = \frac{M_t - M_{t-1}}{P_t} = \frac{M_t - M_{t-1}}{M_{t-1}} \cdot \frac{M_{t-1}}{P_t}$$

$$\pi^{ss} = \theta = \frac{M_t - M_{t-1}}{M_{t-1}} \cancel{=} \cancel{\frac{M^{ss} - M^{ss}}{M^{ss}}}$$

$$\pi_t = \frac{P_t - P_{t-1}}{P_{t-1}} = \cancel{\frac{P^{ss} - P^{ss}}{P^{ss}}}$$

$$\tau = \dot{M}_t \cdot \frac{M_{t-1}}{P_t} = \theta \cdot \frac{M_{t-1}}{P_t} = \theta \cdot \frac{M_{t-1}}{P_{t-1}} \cdot \frac{P_{t-1}}{P_t}$$

$$\tau_t = \theta\,\frac{m_{t-1}}{1+\pi_t}$$

$$\tau_t = \theta \frac{m_{t-1}}{1+\pi_t}$$

$$\tau^{ss} = \theta \frac{m^{ss}}{1+\pi^{ss}} = \frac{\theta\, m^{ss}}{1+\theta}$$

$$c^{ss} = f(k^{ss}) - \delta k^{ss}$$

$$f(k^{ss}) + \cancel{\tau^{ss}} + \cancel{(1}-\delta)k^{ss} + \cancel{\frac{m^{ss}}{1+\theta}} = c^{ss} + \cancel{k^{ss}} + \cancel{m^{ss}}$$

$$c^{ss} = f(k^{ss}) - \delta k^{ss}$$

$$c^{ss} = \left[\frac{\alpha\beta}{1+\beta(\delta-1)}\right]^{\frac{\alpha}{1-\alpha}} - \delta\left[\frac{\alpha\beta}{1+\beta(\delta-1)}\right]^{\frac{1}{1-\alpha}}$$

Deep Parameters پارامترهای عمیق

Superneutrality of Money ثی پول

$$f'_k(k^{ss}) = \frac{1}{\beta} - 1 + \delta$$

$$\frac{u'_c(c_{t+1}, m_{t+1})}{u'_c(c_t, m_t)} = \frac{1/\beta}{f'_k(k_t) + 1 - \delta}$$

$$k < k^{ss}$$

$$\frac{1}{\beta} = 1 + r^{ss} = f'_k(k^{ss}) + 1 - \delta = \frac{1+i^{ss}}{1+\pi^{ss}} = \frac{1+i^{ss}}{1+\theta}$$

$$1 + i^{ss} = \frac{1+\theta}{\beta}$$

$$i^{ss} = \frac{1+\theta}{\beta} - 1$$

$$2^{.55} = \frac{1+\theta}{\beta} - 1$$