Session02

Tuesday, October 17, 2023 9:28 AM

$$\frac{P_t}{P_{t-1}} = 1+\pi_t \qquad \frac{(1+i_{t-1})B_{t-1}}{P_t} = \frac{(1+i_{t-1})\,b_{t-1}}{1+\pi_t}$$

$$\text{Max} \sum_{t=0}^{\infty} \beta^t U(c_t, m_t)$$

s.t.

$$\omega_t = f\left(\frac{k_{t-1}}{1+n}\right) + \tau_t + \left(\frac{1-\delta}{1+n}\right)k_{t-1} + \frac{(1+i_{t-1})b_{t-1} + m_{t-1}}{(1+\pi_t)(1+n)}$$

$$= c_t + k_t + m_t + b_t$$

$$\mathcal{L} = \sum_{t=0}^{\infty} E_t \beta^t \Big\{ U(c_t, m_t) + \lambda_t \Big[f\left(\frac{k_{t-1}}{1+n}\right) + \tau_t + \left(\frac{1-\delta}{1+n}\right)k_{t-1}$$

$$+ \frac{(1+i_{t-1})b_{t-1} + m_{t-1}}{(1+\pi_t)(1+n)} - c_t - k_t - m_t - b_t \Big] \Big\}$$

$$\frac{\partial \mathcal{L}}{\partial c_t} = U'_c(c_t, m_t) - \lambda_t = 0 \qquad (1)$$

$$\frac{\partial \mathcal{L}}{\partial m_t} = U'_m(c_t, m_t) - \lambda_t + \beta E_t \frac{\lambda_{t+1}}{(1+\pi_{t+1})(1+n)} = 0 \qquad (2)$$

$$\frac{\partial \mathcal{L}}{\partial b_t} = -\lambda_t + \beta E_t \frac{\lambda_{t+1}(1+i_t)}{(1+\pi_{t+1})(1+n)} = 0 \qquad (3)$$

$$\frac{\partial \mathcal{L}}{\partial k_t} = -\lambda_t + \frac{\beta}{1+n} E_t \lambda_{t+1}\left[f'\left(\frac{k_t}{1+n}\right) + (1-\delta)\right] = 0 \qquad (4)$$

$$\lim_{t\to\infty} \beta^t \lambda_t x_t = 0 \quad \text{for } x = k, b, m$$

Transversality Condition

$$\frac{\partial \mathcal{L}}{\partial \lambda_t} = 0 \qquad (5)$$

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(3) $$\lambda_t = \beta E_t \frac{\lambda_{t+1}(1+i_t)}{(1+\Pi_{t+1})(1+n)}$$

$$\beta E_t \frac{\lambda_{t+1}}{(1+\Pi_{t+1})(1+n)} = \frac{\lambda_t}{1+i_t}$$

(2) $$u'_m(c_t, m_t) = \lambda_t - \beta E_t \frac{\lambda_{t+1}}{(1+\Pi_{t+1})(1+n)}$$

(3), (2) $\Rightarrow$ $$u'_m(c_t, m_t) = \lambda_t - \frac{\lambda_t}{1+i_t} = \left(\frac{i_t}{1+i_t}\right)\lambda_t$$

(1), (2), (3) $\Rightarrow$ $$\frac{u'_m(c_t, m_t)}{u'_c(c_t, m_t)} = \frac{i_t}{1+i_t}$$

(1), (3) $\Rightarrow$ $$u'_c(c_t, m_t) = \beta E_t \frac{(1+i_t)\, u'_c(c_{t+1}, m_{t+1})}{(1+\Pi_{t+1})(1+n)}$$

(1), (4) $\Rightarrow$ $$u'_c(c_t, m_t) = \frac{\beta}{1+n} E_t\, u'_c(c_{t+1}, m_{t+1})\left[f'\left(\frac{k_t}{1+n}\right) + (1-\delta)\right]$$

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(1), (3), (4) $\Rightarrow$

$$E_t \frac{1+i_t}{1+\Pi_{t+1}} = \left[f'\left(\frac{k_t}{1+n}\right) + (1-\delta)\right]$$

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