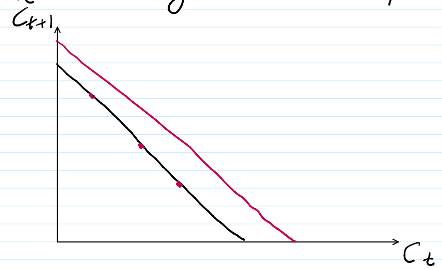
Session03

Sunday, November 5, 2023 10:55 AM

Chapter 2:

Infinite-Horizon and Overlapping-Generations Models:

The Ramsey-Cass-Koopmans Model



Assumptions:

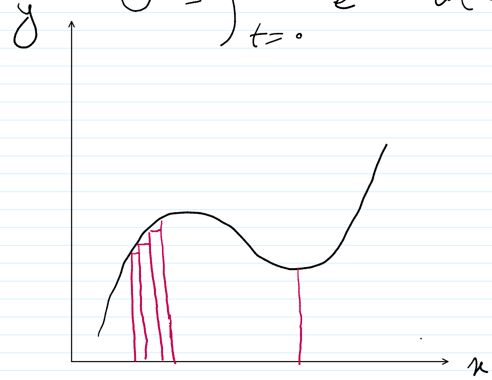
$\delta = 0$ $\quad \Delta \quad\quad T$

$\sigma \quad \Sigma \quad\quad \tau$

$$\dot{K}(t) = Y(t) - \zeta(t)$$

Households:

$$U = \int_{t=0}^{\infty} e^{-\rho t} u(C(t)) \frac{L(t)}{H} dt$$



$\frac{1}{1+r}$ عامل تنزیل

Discount Factor

r نرخ تنزیل

$$\lim_{n \to \infty} \frac{1}{(1+\frac{r}{n})^{tn}} = \lim_{n \to \infty} (1+\frac{r}{n})^{-tn} = e^{-rt}$$

$$u(C(t)) = \frac{C(t)^{1-\theta}}{1-\theta}, \quad \theta > 0, \quad \rho - n - (1-\theta)g > 0$$

CRRA

Constant Relative Risk Aversion

تابع مطلوبیت با ریسک گریزی نسبی ثابت

$$-\frac{C\,u''(C)}{u'(C)} = \theta$$

$$\frac{\qquad}{u'(c)} = \theta$$

$$\theta = 1 \Rightarrow \frac{C(t)^{1-1}}{1-1} = \frac{1}{0} = \infty$$

$$\lim_{\theta \to 1} \frac{C(t)^{1-\theta}}{1-\theta} = \ln C(t)$$

$$r(t) = f'(k(t))$$

پرداختی به سرمایه : $r(t)\,k(t) = k(t)\, f'(k(t))$

$$W(t) = A(t)\left[f(k(t)) - k(t)\, f'(k(t)) \right]$$

اقتصاد را بینسون گروزنه

endowment

$$w(t) = \left[f(k(t)) - k(t)\, f'(k(t)) \right]$$

Households' Budget Constraint:

$$R(t) = \int_{\tau=0}^{t} r(\tau)\, d\tau$$

$$e^{-R(t)}$$

$$\int_{t=0}^{\infty} e^{-R(t)} \frac{c(t)\, L(t)}{H}\, dt \le \frac{K(0)}{H} + \int_{t=0}^{\infty} e^{-R(t)} \frac{W(t)\, L(t)}{H}\, dt$$

$$\lim_{s \to \infty} \left(\frac{K(0)}{H} + \int_{t=0}^{s} e^{-R(t)} \left[\frac{W(t)\, L(t)}{H} - \frac{c(t)\, L(t)}{H} \right] dt \right) \ge 0$$

$$\frac{K(s)}{H} = e^{R(s)} \frac{K(0)}{H} + \int_{t=0}^{s} e^{R(s) - R(t)} \left[W_t - C(t) \right] \frac{L(t)}{H}\, dt$$

طرفین ضرب در $e^{-R(S)}$

$$\lim_{s \to \infty} \frac{e^{-R(S)} K(S)}{H} \ge 0$$

no-Ponzi Game Condition

Households' Maximization Problem:

Households' Maximization Problem:

$$\frac{C(t)^{1-\theta}}{1-\theta} = \frac{\left[A(t)\,c(t)\right]^{1-\theta}}{1-\theta} = \frac{\left[A(0)\,e^{gt}\right]^{1-\theta} c(t)^{1-\theta}}{1-\theta}$$

$$c(t) = \frac{C(t)}{A(t)} \Rightarrow C(t) = A(t)\,c(t)$$

$$A(t) = A(0)\,e^{gt} \qquad L(t) = L(0)\,e^{nt}$$

$$\frac{A(0)^{1-\theta}\, e^{(1-\theta)gt}\, c(t)^{1-\theta}}{1-\theta}$$

$$U = \int_{t=0}^{\infty} e^{-\rho t}\,\frac{C(t)^{1-\theta}}{1-\theta}\,\frac{L(t)}{H}\,dt$$

$$= \int_{t=0}^{\infty} e^{-\rho t}\,\frac{A(0)^{1-\theta}\, e^{(1-\theta)gt}\, c(t)^{1-\theta}}{1-\theta}\,\frac{L(0)\,e^{nt}}{H}\,dt$$

$$= \frac{A(0)^{1-\theta} L(0)}{H}\int_{t=0}^{\infty} e^{-\rho t}\, e^{((1-\theta)g+n)t}\,\frac{c(t)^{1-\theta}}{1-\theta}\,dt$$

$$= B\int_{t=0}^{\infty} e^{-\beta t}\,\frac{c(t)^{1-\theta}}{1-\theta}\,dt$$

$$\beta = \rho - (1-\theta)g - n > 0$$

$$\rho > (1-\theta)g + n$$

$$\int_{t=0}^{\infty} e^{-R(t)}\, c(t)\,\frac{A(t)L(t)}{H}\,dt \le k(0)\,\frac{A(0)L(0)}{H} + \int_{t=0}^{\infty} e^{-R(t)}\, w(t)\,\frac{A(t)L(t)}{H}\,dt$$

$$A(t)L(t) = A(0)\,e^{gt}\,L(0)\,e^{nt} = A(0)L(0)\,e^{(g+n)t}$$

[illegible]

$$\int_{t=0}^{\infty} e^{-R(t)} c(t)\, e^{(n+g)t}\, dt \leq k(0) + \int_{t=0}^{\infty} e^{-R(t)} w(t)\, e^{(n+g)t}\, dt$$

$$\lim_{s \to \infty} e^{-R(s)} e^{(n+g)s} k(s) \geq 0$$

$$\text{Max} \quad U = B \int_{t=0}^{\infty} e^{-\beta t} \frac{c(t)^{1-\theta}}{1-\theta}\, dt$$

s.t.

$$\int_{t=0}^{\infty} e^{-R(t)} c(t)\, e^{(n+g)t}\, dt = k(0) + \int_{t=0}^{\infty} e^{-R(t)} w(t)\, e^{(n+g)t}\, dt$$

$$\mathcal{L} = B \int_{t=0}^{\infty} e^{-\beta t} \frac{c(t)^{1-\theta}}{1-\theta}\, dt + \lambda \left[k(0) + \int_{t=0}^{\infty} e^{-R(t)} w(t)\, e^{(n+g)t}\, dt - \int_{t=0}^{\infty} e^{-R(t)} c(t)\, e^{(n+g)t}\, dt \right]$$