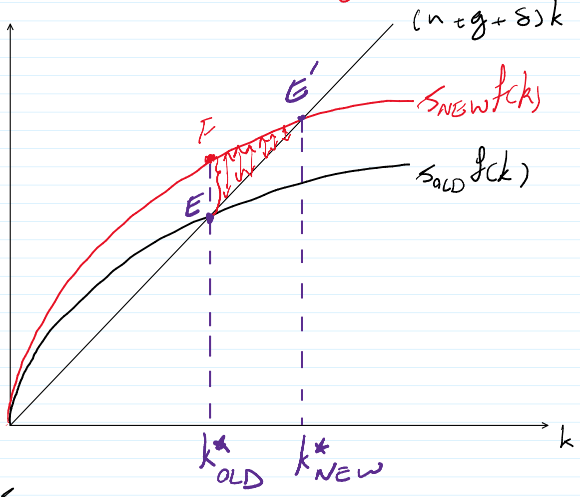
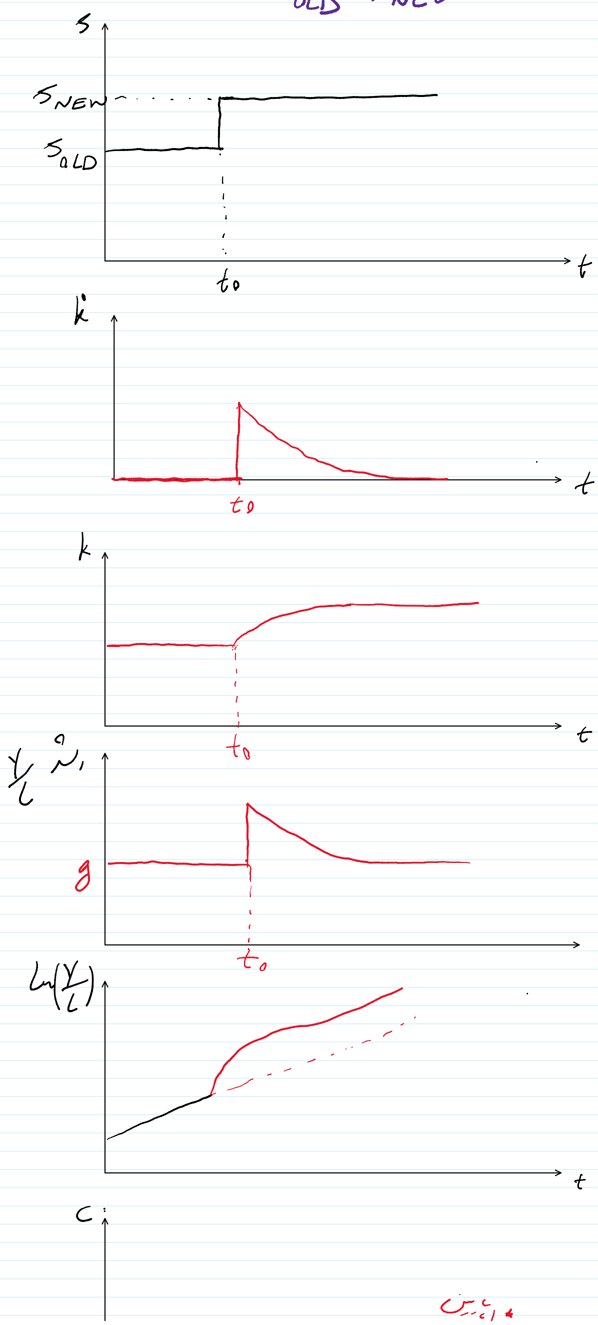
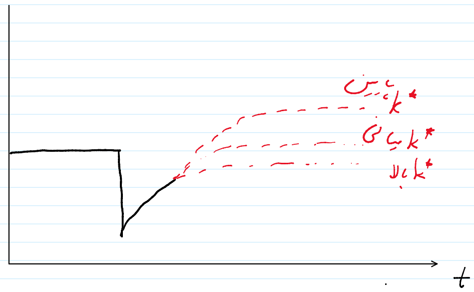
The Impact of a change in the Saving Rate:



$s_{NEW} > s_{OLD}$

نرخ پس انداز





$s(t) = s f(k(t))$

$s^* = s f(k^*)$

$y^* = c^* + s^* = f(k^*) = c^* + s f(k^*)$

$c^* = f(k^*) - s f(k^*)$

$s f(k^*) = (n+g+\delta) k^*$

$c^* = f(k^*) - (n+g+\delta) k^*$

$c^* = f(k^*(s,n,g,\delta)) - (n+g+\delta) k^*(s,n,g,\delta)$

$\frac{\partial c^*}{\partial s} = f'(k^*(s,n,g,\delta)) \frac{\partial k^*}{\partial s} - (n+g+\delta) \frac{\partial k^*}{\partial s}$ $\left\{ \begin{array}{l} > 0 \\ = 0 \\ < 0 \end{array} \right.$

$\frac{\partial c^*}{\partial s} = (\underset{\oplus}{f'(k^*)} - \underset{\oplus}{(n+g+\delta)}) \underset{\oplus}{\frac{\partial k^*}{\partial s}}$

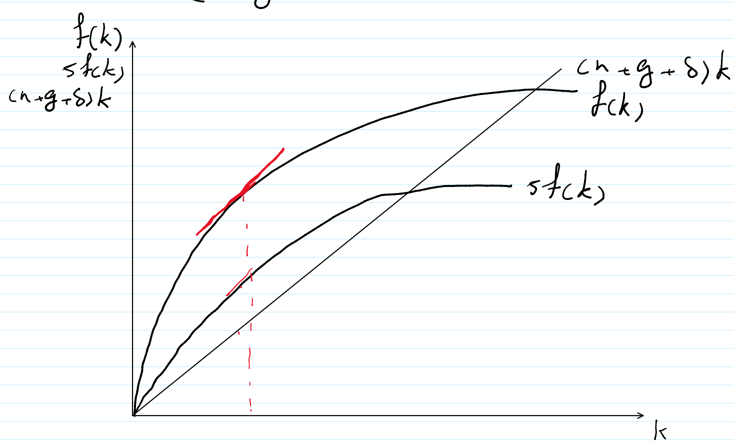
$s f(k^*) = (n+g+\delta) k^*$

$f(k^*) = k^{*\alpha}$

$s k^{*\alpha} = (n+g+\delta) k^*$

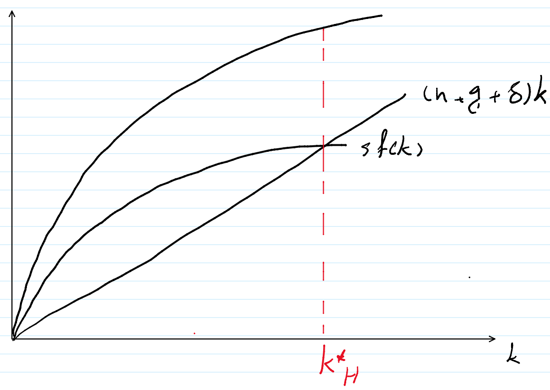
$k^{*1-\alpha} = \frac{s}{n+g+\delta}$

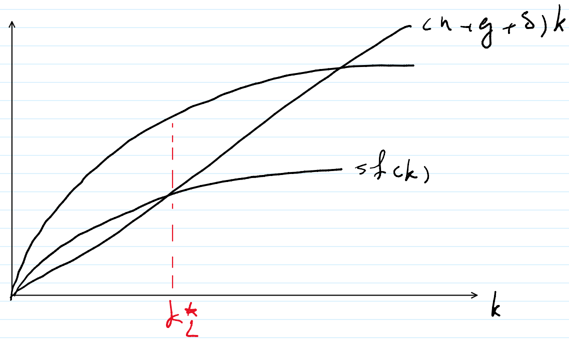
$k^* = \left(\frac{s}{n+g+\delta}\right)^{\frac{1}{1-\alpha}}$

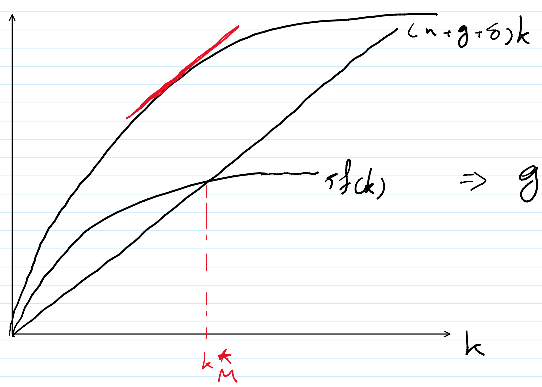


$\frac{\partial c^*}{\partial s} = (f'(k^*) - (n+g+\delta)) \underset{\oplus}{\frac{\partial k^*}{\partial s}}$

$$\frac{\partial c^*}{\partial s} = \left(f(k^*) - (n+g+\delta)\right)\underbrace{\frac{\partial k^*}{\partial s}}_{+}$$







$\Rightarrow$ golden Rule

$$\frac{\partial y^*}{\partial s} = ?$$

$$y^* = f(k^*) = f(k^*(n,g,\delta,s))$$

$$\frac{\partial y^*}{\partial s} = f'(k^*)\frac{\partial k^*(n,g,\delta,s)}{\partial s}$$

$\dot{k} = 0 \Rightarrow k^*$ $\quad \dot{k}(t) = s f(k^*(n,g,\delta,s)) - (n+g+\delta)k^*(n,g,\delta,s) = 0$

$$s f'(k^*)\frac{\partial k^*(n,g,\delta,s)}{\partial s} + f(k^*) = (n+g+\delta)\frac{\partial k^*(n,g,\delta,s)}{\partial s}$$

$\Rightarrow \partial k^* \quad - f'(k^*)\, \partial k^* \quad f(k^*)$

$$(n+g+\delta)\frac{\partial k^*}{\partial s} - s f'(k^*)\frac{\partial k^*}{\partial s} = f(k^*)$$

$$\frac{\partial k^*}{\partial s} = \frac{f(k^*)}{(n+g+\delta) - s f'(k^*)}$$

$$\frac{\partial y^*}{\partial s} = f'(k^*)\frac{\partial k^*}{\partial s} = \frac{f'(k^*)\, f(k^*)}{(n+g+\delta) - s f'(k^*)}$$

$$\frac{s}{y^*}\frac{\partial y^*}{\partial s} = \frac{s}{f(k^*)}\cdot\frac{f'(k^*)\, f(k^*)}{(n+g+\delta) - s f'(k^*)}$$

$$= \frac{s f(k^*)\cdot f'(k^*)}{f(k^*)(n+g+\delta) - s f'(k^*) f(k^*)}$$

$$s f(k^*) = (n+g+\delta) k^*$$

$$\frac{s}{y^*}\frac{\partial y^*}{\partial s} = \frac{(n+g+\delta) k^* f'(k^*)}{f(k^*)(n+g+\delta) - (n+g+\delta) k^* f'(k^*)}$$

$$= \frac{(n+g+\delta) k^* f'(k^*)}{f(k^*)\left[(n+g+\delta) - (n+g+\delta)\frac{k^* f'(k^*)}{f(k^*)}\right]}$$

کشش تولید نسبت به سرمایه

$$\frac{k^*}{y^*}\cdot\frac{\partial y^*}{\partial k^*} = \frac{k^*}{f(k^*)}\cdot f'(k^*) = \alpha_k(k^*)$$

$$f(k^*) = k^{*\alpha}$$

$$\frac{k^*}{y^*}\cdot\frac{\partial y^*}{\partial k^*} = \frac{d\ln y^*}{d\ln k^*} \qquad y^* = f(k^*) = k^{*\alpha}$$

$$\underbrace{\ln y^*}_{\text{[illegible]}} = \alpha\underbrace{\ln k^*}_{\text{[illegible]}} \qquad \frac{d\ln y^*}{d\ln k^*} = \frac{d\ \text{[illegible]}}{d\ \text{[illegible]}} = \alpha$$

$$\frac{s}{\ }\cdot\frac{\partial y^*}{\ } = \frac{\alpha_k(k^*)}{\ }$$

$$\frac{s}{y^*}\cdot\frac{\partial y^*}{\partial s} = \frac{\alpha_k(k^*)}{1-\alpha_k(k^*)}$$

در حالت خاص $\quad \frac{s}{y^*}\cdot\frac{\partial y^*}{\partial s} = \frac{\alpha}{1-\alpha}$

tavakolianh.github.io

The Speed of Convergence:

$$\dot{k} \qquad\qquad (k-k^*)$$

$$\dot{k} = s f(k) - (n+g+\delta)k$$

$$f(x) \simeq \frac{f(x_0)}{0!} + \frac{f'(x_0)}{1!}(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \cdots + \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n$$

$$f(x) \simeq f(x_0) + f'(x_0)(x-x_0)$$

$$\dot{k}(t) = s f(k(t)) - (n+g+\delta)k(t)$$

$$\dot{k}(t) \simeq \left[\frac{\partial \dot{k}(k)}{\partial k}\bigg|_{k=k^*}\right](k-k^*)$$

$$\dot{k}(t) \simeq -\lambda(k-k^*)$$

$$\lambda = -\left[\frac{\partial \dot{k}(k)}{\partial k}\bigg|_{k=k^*}\right] \qquad \text{سرعت همگرایی}$$

$$\dot{k}(t) = \frac{dk(t)}{dt} \simeq -\lambda(k-k^*)$$

$$k(t) \simeq k^* + e^{-\lambda t}[k(0)-k^*]$$

$$\lambda = -\frac{\partial \dot{k}}{\partial k}\bigg|_{k=k^*} = -[s f'(k^*) - (n+g+\delta)]$$

f' , k^*

$$\lambda = - \frac{\partial \dot{k}}{\partial k}\Big|_{k=k^*} = -[s f'(k^*) - (n+g+\delta)]$$

$$= (n+g+\delta) - s f'(k^*)$$

$$k^* \Rightarrow \dot{k} = 0 \Rightarrow s f(k^*) = (n+g+\delta) k^*$$

$$s = \frac{(n+g+\delta) k^*}{f(k^*)}$$

$$\lambda = (n+g+\delta) - \frac{k^* f'(k^*)}{f(k^*)} (n+g+\delta)$$

$$\lambda = \Big(1 - \underbrace{\frac{k^* f'(k^*)}{f(k^*)}}_{\alpha_k(k^*)}\Big)(n+g+\delta)$$

$$\lambda = (1 - \alpha_k(k^*))(n+g+\delta)$$

Growth Accounting:

$$Y(t) = F(K(t), A(t) L(t))$$

$$\dot{Y}(t) = \frac{\partial Y(t)}{\partial K(t)} \cdot \dot{K}(t) + \frac{\partial Y(t)}{\partial L(t)} \cdot \dot{L}(t) + \frac{\partial Y(t)}{\partial A(t)} \cdot \dot{A}(t)$$

$$\frac{\dot{Y}(t)}{Y(t)} = \frac{\partial Y(t)}{\partial K(t)} \cdot \frac{\dot{K}(t)}{K(t)} \cdot \frac{K(t)}{Y(t)} + \frac{\partial Y(t)}{\partial L(t)} \cdot \frac{\dot{L}(t)}{L(t)} \cdot \frac{L(t)}{Y(t)}$$

$$+ \boxed{\frac{\partial Y(t)}{\partial A(t)} \cdot \frac{\dot{A}(t)}{A(t)} \cdot \frac{A(t)}{Y(t)}} \quad R(t)$$

$$\frac{\dot{Y}(t)}{Y(t)} = \alpha_k(t) \frac{\dot{K}(t)}{K(t)} + \alpha_L(t) \frac{\dot{L}(t)}{L(t)} + R(t)$$

$$\alpha_k(t) + \alpha_L(t) = 1$$

$$\frac{\dot{Y}(t)}{Y(t)} = \alpha_k(t) \frac{\dot{K}(t)}{K(t)} + (1 - \alpha_k(t)) \frac{\dot{L}(t)}{L(t)} + R(t)$$

$$\frac{\dot{Y}(t)}{Y(t)} = \alpha_K(t) \frac{\dot{K}(t)}{K(t)} + (1-\alpha_K(t)) \frac{\dot{L}(t)}{L(t)} + R(t)$$

$$\underbrace{\frac{\dot{Y}(t)}{Y(t)} - \frac{\dot{L}(t)}{L(t)}}_{\text{رشد تولید سرانه}} = \alpha_K(t) \Bigg[\underbrace{\frac{\dot{K}(t)}{K(t)} - \frac{\dot{L}(t)}{L(t)}}_{\text{رشد سرمایه سرانه}} \Bigg] + R(t)$$

Solow Risidual

پسماند سولو

Natural Resources and Land: A Baseline Case

$$Y(t) = K(t)^{\alpha} R(t)^{\beta} T(t)^{\gamma} \left[A(t) L(t) \right]^{1-\alpha-\beta-\gamma}$$

که منابع طبیعی

زمین

$$\alpha > 0 \ , \ \beta > 0 \ , \ \gamma > 0 \qquad \alpha + \beta + \gamma < 1$$

$$\dot{A}(t) = g A(t) \ , \quad \dot{L}(t) = n L(t)$$

$$\dot{K}(t) = s Y(t) - \delta K(t)$$

$$\dot{T}(t) = 0$$

$$\dot{R}(t) = -b R(t), \qquad b > 0$$

$$\frac{\dot{K}(t)}{K(t)} = s \frac{Y(t)}{K(t)} - \delta$$

$$\ln Y(t) = \alpha \ln K(t) + \beta \ln R(t) + \gamma \ln T(t)$$
$$+ (1-\alpha-\beta-\gamma) \left[\ln A(t) + \ln L(t) \right]$$

$$g_Y(t) = \alpha g_K(t) + \beta \overbrace{g_R(t)}^{-b} + \gamma \overbrace{g_T(t)}^{\text{صفر}}$$

$$g_Y(t) = \alpha\, g_K(t) + \beta\, g_R(t) + \gamma\, g_T(t)$$

$$+ (1-\alpha-\beta-\gamma)\left[\underbrace{g_L(t)}_{n} + \underbrace{g_A(t)}_{g}\right]$$

$$g_Y(t) = \alpha\, g_K(t) - \beta b + (1-\alpha-\beta-\gamma)(n+g)$$

مسیر رشد متوازن

$$g_Y = g_K = \cdots$$

$$g_Y^{bgp} = \alpha\, g_Y^{bgp} - \beta b + (1-\alpha-\beta-\gamma)(n+g)$$

$$g_Y^{bgp} = \frac{(1-\alpha-\beta-\gamma)(n+g) - \beta b}{1-\alpha}$$

$$g_{Y/L}^{bgp} = g_Y^{bgp} - g_L^{bgp} = g_Y^{bgp} - n$$

$$= \frac{(1-\alpha-\beta-\gamma)(n+g) - \beta b}{1-\alpha} - n$$

$$= \frac{(1-\alpha-\beta-\gamma)\,g - \beta b - (\beta+\gamma)\,n}{1-\alpha}$$