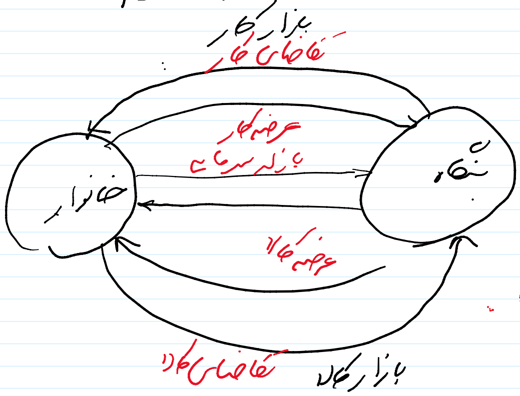
Session01

Tuesday, September 26, 2023 11:20 AM

$$Y = C + I$$
$$Y = C + S$$



$$\cancel{C} + I = \cancel{C} + S$$
$$I = S$$

$$g_t = \frac{Y_t - Y_{t-1}}{Y_{t-1}}$$

$$K_t = I_t + K_{t-1} - \delta K_{t-1} \qquad \sigma \quad \delta \quad \Sigma \quad \Delta$$

$$K_t = (1-\delta) K_{t-1} + I_t \Rightarrow Y_t = F(K_{t-1}, L_t)$$

انتهای $t-1$ انتهای t

ابتدای t ابتدای $t+1$

دوره $t-1$ دوره t

$$K_{t+1} = (1-\delta) K_t + I_t \qquad Y_t = F(K_t, L_t)$$

$$K_{t+1}$$

$I \; S$

The Solow Growth Model:

Assumptions:

$$Y(t) = C(t) + I(t)$$
$$Y(t) = C(t) + S(t)$$

$$Y(t) = F(K(t), A(t)L(t))$$

تکنولوژی   تابع تولید کارافزا

$$Y(t) = F(A(t)K(t),\ L(t))$$

$$Y(t) = A(t)F(K(t), L(t))$$

Hicks-neutral

$$Y(t) = F(A(t)K(t),\ A(t)L(t))$$

تابع تولید همگن از درجه یک است.

$$F(xK, xAL)$$

$$= xF(K, AL) = xY$$

$$x = \frac{1}{AL}$$

$$F\left(\frac{K}{AL}, \frac{AL}{AL}\right) = F\left(\frac{K}{AL}, 1\right)$$

$$= \frac{1}{AL}F(K, AL) = \frac{Y(t)}{A(t)L(t)} = y(t)$$

$$y(t) = F\left(\frac{K(t)}{A(t)L(t)}, 1\right) = f(k(t))$$

$$y(t) = \frac{Y(t)}{A(t)L(t)}$$ تولید سرانه موثر

$$f(k(t)) = F\left(\frac{K(t)}{A(t)L(t)}, 1\right)$$

$$k(t) = \frac{K(t)}{A(t)L(t)}$$ سرمایه سرانه موثر

$$y(t) = f(k(t))$$

وقتی تولید همگن از درجه یک باشد (بازده ثابت نسبت به مقیاس) کل تولید بین عوامل تولید توزیع می‌شود.

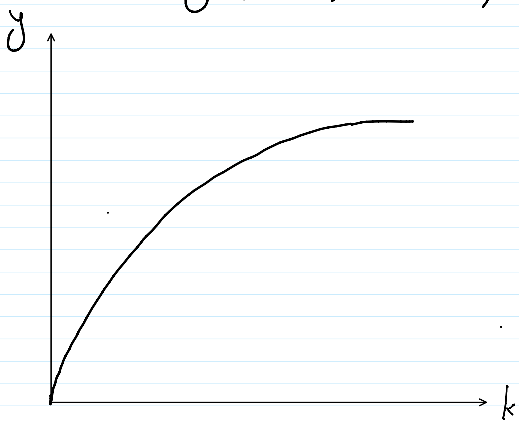
$$Y(t) = W(t)L(t) + R(t)K(t)$$

$$Y(t) = F'_L(t)L(t) + F'_K(t)K(t)$$

خب سود صفر

$$\pi(t) = Y(t) - \left(w(t)L(t) + R(t)K(t)\right) = 0$$

$$y(t) = f(k(t))$$



$$f(0) = 0 \qquad f'(k) > 0 \qquad f''(k) < 0$$

Inada Conditions

$$\lim_{k \to 0} f'(k) = \infty \qquad \lim_{k \to \infty} f'(k) = 0$$

$$Y(t) = F(K(t), A(t)L(t)) = K(t)^{\alpha}(A(t)L(t))^{1-\alpha}$$

$$\frac{Y(t)}{A(t)L(t)} = \left(\frac{K(t)}{A(t)L(t)}\right)^{\alpha}(1)^{1-\alpha}$$

$$y(t) = k(t)^{\alpha}$$

CES

$$Y = \left[\alpha K^{\theta} + (1-\alpha)(AL)^{\theta}\right]^{\frac{1}{\theta}}$$

$$\frac{\theta}{1-\theta}$$

$$Y = \min\left(\frac{K}{\alpha}, \frac{AL}{1-\alpha}\right)$$

$X(t)$

$$dX(t) \Rightarrow \frac{dX(t)}{dt} = \dot{X}(t)$$

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$$\Delta X_t = X_t - X_{t-1}$$

$$\frac{\Delta X_t}{\Delta t} = \frac{X_t - X_{t-1}}{t-(t-1)} = X_t - X_{t-1} = \Delta X_t$$

$$\frac{\dot{X}(t)}{X(t)} =$$ نرخ رشد $X(t)$

$$\dot{L}(t) = \frac{dL(t)}{dt} = nL(t)$$ فرض

$$\frac{\dot{L}(t)}{L(t)} = n$$ پارامتر

نرخ رشد نیروی کار ثابت و برابر n است

$$\frac{\dot{A}(t)}{A(t)} = g$$

$$n = \frac{\dot{L}(t)}{L(t)} = \frac{d\ln L(t)}{dt} = \frac{\frac{dL(t)}{dt}}{L(t)}$$

$$L(t) = L(0)\,e^{nt}$$

$$A(t) = A(0)\,e^{gt}$$

$$L_t = L_0(1+n)^t$$

$$A_t = A_0(1+g)^t$$

$$\ln L(t) = \ln L(0) + nt$$

$$\ln A(t) = \ln A(0) + gt$$

$$K_t = (1-\delta)K_{t-1} + I_t$$

$$\Delta K_t = K_t - K_{t-1} = I_t - \delta K_{t-1}$$

$$\dot{K}(t) = \frac{dK(t)}{dt} = I(t) - \delta K(t)$$

$$Y(t) = C(t) + I(t)$$

$$I(t) = Y(t) - C(t)$$

$$Y(t) = C(t) + I(t)$$
$$I(t) = Y(t) - C(t)$$
$$\dot{K}(t) = Y(t) - C(t) - \delta K(t)$$
$$Y(t) = C(t) + S(t)$$ پس انداز
$$S(t) = Y(t) - C(t)$$
$$\dot{K}(t) = S(t) - \delta K(t)$$

قید بودجه خانوار

$$C(t) + S(t) = W(t)L(t) + R(t)K(t) = Y(t)$$
$$C(t) + I(t) = \quad \sim \quad \sim \quad \sim$$
$$S(t) = I(t)$$
$$\dot{K}(t) = I(t) - \delta K(t) \Rightarrow I(t) = \dot{K}(t) + \delta K(t)$$
$$C(t) + \dot{K}(t) + \delta K(t) = W(t)L(t) + R(t)K(t) = Y(t) = F(K(t), A(t)L(t))$$
$$\dot{K}(t) + \delta K(t) = Y(t) - C(t) = S(t)$$

$$S(t) = s\,Y(t) \qquad 0 < s \leq 1$$

نرخ پس انداز

$$\dot{K}(t) = s\,Y(t) - \delta K(t)$$

The Dynamics of the model:

$$k(t) = \frac{K(t)}{A(t)L(t)}$$

$$\dot{k}(t) = \frac{dk(t)}{dt} = \frac{\dot{K}(t)A(t)L(t) - K(t)\left(\dot{A}(t)L(t) + A(t)\dot{L}(t)\right)}{\left(A(t)L(t)\right)^2}$$

$$\dot{k}(t) = \frac{\dot{K}(t)}{A(t)L(t)} - \frac{K(t)}{A(t)L(t)}\left(\frac{\dot{A}(t)\cancel{L(t)}}{A(t)\cancel{L(t)}} + \frac{\cancel{A(t)}\dot{L}(t)}{\cancel{A(t)}L(t)}\right)$$

$$\dot{k}(t) = \frac{\dot{K}(t)}{A(t)L(t)} - \underbrace{\frac{K(t)}{A(t)L(t)}}_{k(t)}\left(\underbrace{\frac{\dot{A}(t)\cancel{L(t)}}{A(t)\cancel{L(t)}}}_{g} + \underbrace{\frac{\cancel{A(t)}\dot{L}(t)}{\cancel{A(t)}L(t)}}_{n}\right)$$

$$\dot{k}(t) = \frac{sY(t) - \delta K(t)}{A(t)L(t)} - (n+g)k(t)$$

$$\dot{k}(t) = s\,y(t) - \delta k(t) - (n+g)k(t)$$

$$\frac{dk(t)}{dt} = \dot{k}(t) = s\,f(k_t) - (n+g+\delta)k(t) = 0$$

معادله کلیدی مدل رشد سولو

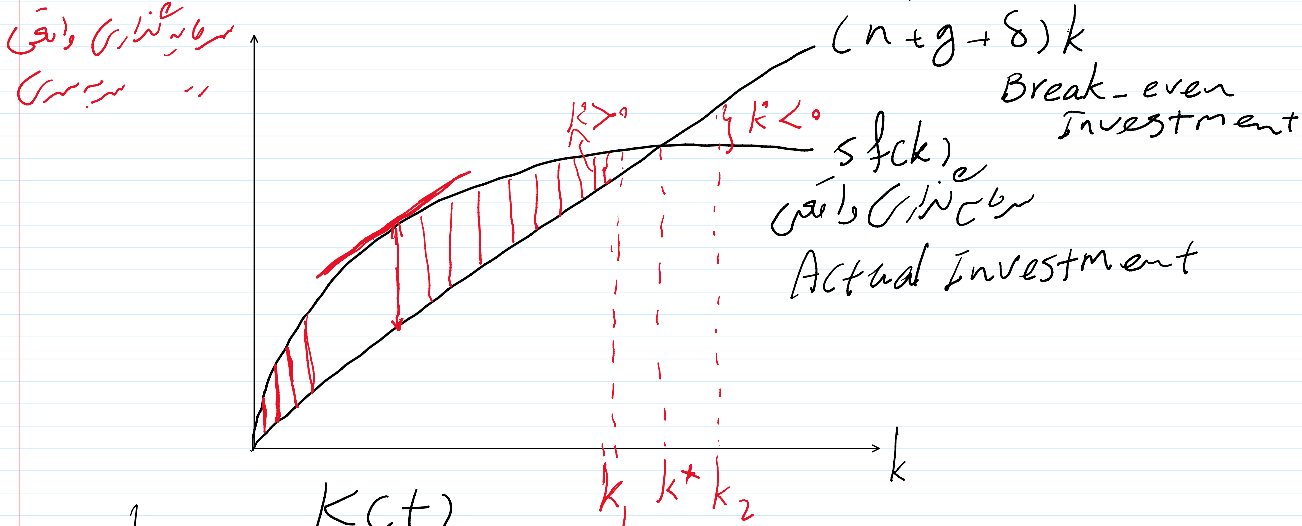
Steady State

وضعیت پایدار

$$X_{t-1} = X_t = E_t\,X_{t+1} = \bar{X}$$

$$\dot{k}(t) = 0 \Rightarrow \underbrace{s\,f(k(t))}_{\text{پس انداز سرانه موثر}} = (n+g+\delta)k(t)$$

پس انداز سرانه موثر = سرمایه گذاری سرانه موثر



$$k(t) = \frac{K(t)}{A(t)L(t)}$$

$$k^* = \frac{K^*}{A^*L^*}$$

$$k^* = \frac{K}{A^* L^*}$$

$$s f(k) > (n+g+\delta) k \Rightarrow \dot{k} > 0$$

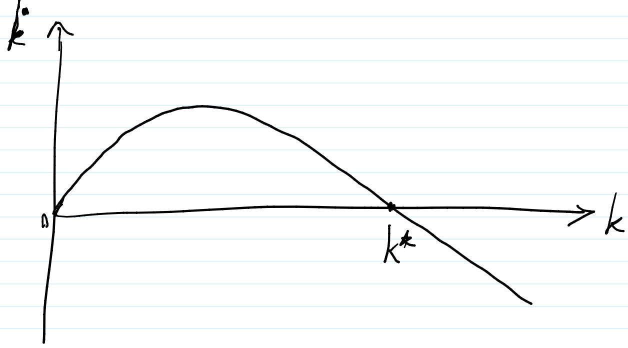
The Balanced Growth Path:

سیر رشد متوازن

$$\dot{k}(t) = s f(k(t)) - (n+g+\delta) k(t)$$

نمودار فازی

phase Diagram



$$\dot{k}(t) = 0 \qquad k(t) = \frac{K(t)}{A(t)L(t)} \Rightarrow \ln k(t) = \ln K(t) - \ln A(t) - \ln L(t)$$

$$\frac{\dot{k}(t)}{k(t)} = \frac{\dot{K}(t)}{K(t)} - \underbrace{\frac{\dot{A}(t)}{A(t)}}_{g} - \underbrace{\frac{\dot{L}(t)}{L(t)}}_{n} = 0$$

$$k^* \Rightarrow \frac{\dot{K}(t)}{K(t)} = n+g$$

در تعادل نرخ رشد سرمایه برابر با جمع نرخ رشد نیروی کار و تکنولوژی است.

$$Y(t) = K(t)^{\alpha} \left(A(t)L(t)\right)^{1-\alpha}$$

$$\ln Y(t) = \alpha \ln K(t) + (1-\alpha)(\ln A(t) + \ln L(t))$$

$$\frac{\dot{Y}(t)}{Y(t)} = \alpha \underbrace{\frac{\dot{K}(t)}{K(t)}}_{(n+g)} + (1-\alpha)\underbrace{\left(\frac{\dot{A}(t)}{A(t)} + \frac{\dot{L}(t)}{L(t)}\right)}_{n+g}$$

$(n+g) \qquad\qquad n+g$

$$\frac{\dot{Y}(t)}{Y(t)} = \alpha(n+g) + (1-\alpha)(n+g) = n+g = \frac{\dot{K}(t)}{K(t)}$$

مسیر رشد متوازن = در تعادل کلیه متغیرهای حقیقی با نرخ رشد یکسان رشد می کند.

$$\frac{\dot{Y}(t)}{Y(t)} = \frac{\hat{C}(t)}{C(t)} = \frac{\dot{I}(t)}{I(t)} = \frac{\dot{K}(t)}{K(t)} = \frac{\dot{S}(t)}{S(t)} = n+g$$

$$\frac{K}{L} = Ak \qquad g_{\frac{K}{L}} = g + \frac{\dot{k}(t)}{k(t)} = g$$

نرخ رشد سرمایه سرانه

صفر

$$g_{\frac{K}{L}} = g_{\frac{Y}{L}} = g_{\frac{C}{L}} = g_{\frac{I}{L}} = g_{\frac{S}{L}} = g$$