Uhlig's Toolkit

SFU

$$Y = C + I$$
$$Y = C + S$$
$$C + I = C + S$$
$$I = S$$
$$K_t = K_{t-1} + I_t - \delta K_{t-1}$$

$$\text{Max } U(x, y)$$
s.t.
$$x P_x + y P_y = M$$
$$\frac{U'_x}{U'_y} = \frac{P_x}{P_y}$$

مخارج مالیات

قید بودجه خانوار:

$$P_t C_t + P_t I_t + P_t T_t + B_t + M_t = P_t W_t L_t + r_t K_{t-1} P_t$$
$$+ (1 + i_{t-1}) B_{t-1} + M_{t-1} + P_t D_t$$

قید بودجه دولت:

$$P_t G_t + (1 + i_{t-1}) B_{t-1} = P_t T_t + M_t - M_{t-1} + B_t$$

$$P_t C_t + P_t I_t + P_t G_t = P_t w_t L_t + r_t P_t K_{t-1} + P_t D_t$$
$$C_t + I_t + G_t = w_t L_t + r_t K_{t-1} + D_t$$
$$Y_t = w_t L_t + r_t K_{t-1} + D_t = F(K_{t-1}, L_t)$$

$$Y_t = C_t + I_t + G_t$$
$$K_t = (1 - \delta) K_{t-1} + I_t$$

طبقه‌بندی متغیرها:

$$y_t = \alpha + \beta x_t + \varepsilon_t$$
$$y_t = \hat{\alpha} + \hat{\beta} x_t + e_t$$
$$y(t)$$

$$E(\varepsilon_t) = 0$$

متغیرهای تصادفی و قطعی:

متغیرهای درون‌زا و برون‌زا:

$$y_t = \alpha + \beta x_t + \varepsilon_t$$

Control and State variables — متغیرهای کنترل و وضعیت:

هر متغیری که وقفه آن در مدل باشد متغیر وضعیت است مثل K_{t-1}

$$y_t = \alpha + \beta y_{t-1} + \varepsilon_t$$ معادله تفاضلی مرتبه اول (ریاضی)

$$y = a + bx + cx^2$$

(AR(1)) AutoRegression — اقتصاد سنجی: خودرگرسیون

روش حل معادله تفاضلی مرتبه اول:

$$y_t = \alpha + \beta y_{t-1} + \varepsilon_t, \quad E(\varepsilon_t) = 0$$
$$y_{t-1} = \alpha + \beta y_{t-2} + \varepsilon_{t-1}$$

① روش جایگذاری

② روش عملگر وقفه

$$y_t = \alpha + \beta y_{t-1} + \varepsilon_t \,, \qquad E(\varepsilon_t) = 0$$

$$y_{t-1} = \alpha + \beta y_{t-2} + \varepsilon_{t-1}$$

① روش جایگذاری

② روش عملگر وقفه

③ " ضرایب نامعین

$$y_t = \alpha + \beta(\alpha + \beta y_{t-2} + \varepsilon_{t-1}) + \varepsilon_t$$

$$y_t = \alpha + \alpha\beta + \beta^2 y_{t-2} + \varepsilon_t + \beta\varepsilon_{t-1}$$

$$y_{t-2} = \alpha + \beta y_{t-3} + \varepsilon_{t-2}$$

$$\vdots$$

$$y_t = \underbrace{\alpha + \alpha\beta + \alpha\beta^2 + \cdots + \alpha\beta^t}_{\text{پارامترها}} + \underbrace{\beta^{t+1} y_{-1}}_{\text{مقدار اولیه}} + \underbrace{\varepsilon_t + \beta\varepsilon_{t-1} + \cdots + \beta^t\varepsilon_0}_{\text{مقادیر تصادفی حال و گذشته}}$$

$$E(y_t) = \alpha\left[1 + \beta + \beta^2 + \cdots + \beta^t\right] + \beta^{t+1} y_{-1}$$

$$E(y_t) = \alpha\frac{[1-\beta^t]}{1-\beta} + \beta^{t+1} y_{-1}$$

$$\beta \begin{cases} < 1 \\ = 1 \\ > 1 \end{cases}$$

if $\beta < 1$

$$\lim_{t\to\infty} E(y_t) = \lim_{t\to\infty} \frac{\alpha[1-\overset{\text{صفر}}{\beta^t}]}{1-\beta} + \lim_{t\to\infty} \overset{\text{صفر}}{\beta^{t+1}} y_{-1}$$

$$= \frac{\alpha}{1-\beta} + 0$$

$$E(y_t) = \frac{\alpha}{1-\beta}$$

Steady State :

$$X_t = X_{t-1} = E_t X_{t+1} = \bar{X}$$

$$y_t = \alpha + \beta y_{t-1} + \varepsilon_t$$

$$\bar{y} = \alpha + \beta\bar{y} + \cancel{\bar{\varepsilon}} \to 0$$

$$(1-\beta)\bar{y} = \alpha$$

$$\bar{y} = \frac{\alpha}{1-\beta}$$

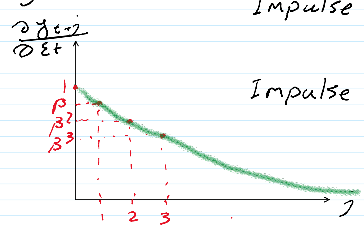
$\beta < 1$

$$y_t = \underbrace{\alpha\left[1 + \beta + \beta^2 + \cdots + \beta^t\right]}_{\frac{\alpha}{1-\beta}} + \underbrace{\overset{\text{صفر}}{\cancel{\beta^{t+1} y_{-1}}}} + \varepsilon_t + \beta\varepsilon_{t-1} + \cdots + \beta^t\varepsilon_0$$

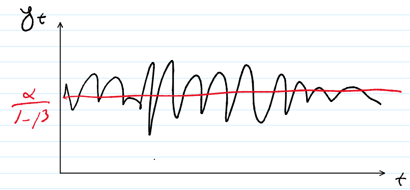
$$y_t = \frac{\alpha}{1-\beta} + \overset{0.01}{\varepsilon_t} + \beta\overset{0.01}{\varepsilon_{t-1}} + \beta^2\overset{0.01}{\varepsilon_{t-2}} + \cdots$$

$$\frac{\partial y_t}{\partial \varepsilon_{t-j}} = \frac{\partial y_{t+j}}{\partial \varepsilon_t} = \beta^j \qquad j \in [0, \infty)$$

Impulse Response



Impulse Response Function



$\beta = 1$

$$\bar{y} = \frac{\alpha}{1-\beta} = \frac{\alpha}{1-1} = \frac{\alpha}{0} = \infty \qquad 1-\beta = 0 \qquad \text{ریشه واحد}$$

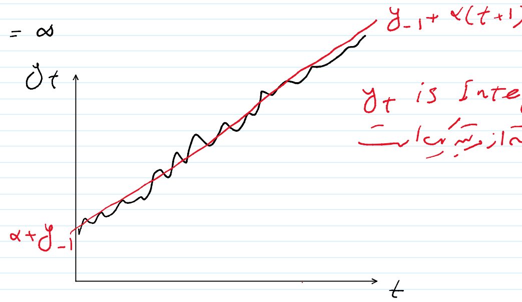
$$y_t = \alpha\left[1+\beta+\beta^2+\cdots+\beta^t\right] + \beta^{t+1} y_{-1} + \varepsilon_t + \beta\varepsilon_{t-1} + \cdots + \beta^t \varepsilon_0$$

$$y_t = \alpha\underbrace{\left[1+1+1+\cdots+1\right]}_{t+1} + y_{-1} + \overset{0.01}{\varepsilon_t} + \overset{0.01}{\varepsilon_{t-1}} + \cdots + \overset{0.01}{\varepsilon_0}$$

$$y_t = \alpha(t+1) + y_{-1} + \varepsilon_t + \varepsilon_{t-1} + \cdots + \varepsilon_0$$

$$E(y_t) = \alpha(t+1) + y_{-1} \leftarrow$$

$$\lim_{t\to\infty} E(y_t) = \infty$$



y_t is Integrated

y_t انباشته از مرتبه یک است

$\beta > 1$

$$\frac{1}{\beta} y_t = \frac{\alpha}{\beta} + \frac{\beta}{\beta} y_{t-1} + \frac{1}{\beta}\varepsilon_t$$

$$\frac{1}{\beta} y_t = \frac{\alpha}{\beta} + y_{t-1} + \frac{1}{\beta}\varepsilon_t$$

$$y_{t-1} = -\frac{\alpha}{\beta} + \frac{1}{\beta} y_t - \frac{1}{\beta}\varepsilon_t$$

$$y_t = -\frac{\alpha}{\beta} + \frac{1}{\beta} y_{t+1} - \frac{1}{\beta}\varepsilon_{t+1}$$

$$y_t = -\frac{\alpha}{\beta} + \frac{1}{\beta}\left[-\frac{\alpha}{\beta} + \frac{1}{\beta} y_{t+2} - \frac{1}{\beta}\varepsilon_{t+2}\right] - \frac{1}{\beta}\varepsilon_{t+1}$$

$\vdots$

$$y_t = -\frac{\alpha}{\beta}\left[1 + \frac{1}{\beta} + \frac{1}{\beta^2} + \cdots\right] + \overset{\text{صفر}}{\cancel{\frac{1}{\beta^t} y_{2t}}} - \frac{1}{\beta}\left[\varepsilon_{t+1} + \frac{1}{\beta}\varepsilon_{t+2} + \cdots\right]$$

$$\beta > 1 \Rightarrow \frac{1}{\beta} < 1$$

$$y_t = \frac{-\frac{\alpha}{\beta}}{1 - \frac{1}{\beta}} - \frac{1}{\beta}\left[\varepsilon_{t+1} + \frac{1}{\beta}\varepsilon_{t+2} + \cdots\right]$$

$$y_t = \frac{\alpha}{1-\beta} - \frac{1}{\beta}\left[\varepsilon_{t+1} + \frac{1}{\beta}\varepsilon_{t+2} + \cdots\right]$$

$$E y_t = \frac{\alpha}{1-\beta}$$

$$\pi_t = \beta E_t \pi_{t+1} + k x_t \qquad \text{منحنی فیلیپس}$$

$$0 < \beta < 1$$

متغیرهای از قبل معین و از قبل نامعین

Predetermined and non-predetermined variables

$$\pi_t = \frac{\beta}{1+\beta} E_t \pi_{t+1} + \frac{1}{1+\beta} \pi_{t-1} + \kappa x_t$$

مدل ساده ادوار تجاری حقیقی (RBC) یا مدل رشد نئوکلاسیکی

کنشگران اقتصادی:

① مصرف کننده (خانوار) : ترجیحات

② تولیدکننده : تولید (تکنولوژی)

③ Endowments

④ اطلاعات

دو رویکرد هدف‌گذاری:

① برنامه‌ریز اجتماعی

② تعادل رقابتی

ترجیحات:

$$U = E\left[\sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\eta} - 1}{1-\eta} - \chi \frac{N_t^{1+\sigma}}{1+\sigma} \right)\right]$$

$\chi = 0$ $\quad 0 < \beta < 1$ عامل تنزیل ذهنی

$\beta = \frac{1}{1+\rho}$ $\quad e^{\beta t}$

$\rho > 0$ نرخ تنزیل ذهنی

η: عکس کشش جانشینی بین دوره‌ای مصرف

$\eta > 0$

$\sigma > 0$ عکس عرضه کار فریش

Frisch

قید بودجه خانوار:

$$C_t + I_t = w_t N_t + R_t K_{t-1}$$

$$Y_t = Z_t K_{t-1}^{\rho} N_t^{1-\rho} \qquad 0 < \rho < 1$$

شرط سود صفر در صنعت بازار

$$\underbrace{N \times MP_N}_{w} + \underbrace{K \times MP_K}_{r} = Y$$

$$w = MP_N \quad , \quad R = MP_K$$

$$\begin{cases} C_t + I_t = Y_t = Z_t K_{t-1}^{\rho} N_t^{1-\rho} \\ K_t = (1-\delta) K_{t-1} + I_t \end{cases}$$

$$I_t = K_t - (1-\delta) K_{t-1}$$

$$C_t + K_t = Z_t K_{t-1}^{\rho} N_t^{1-\rho} + (1-\delta) K_{t-1}$$

$$C_t + \underbrace{K_t - K_{t-1}}_{\Delta K_t} = Z_t K_{t-1}^{\rho} N_t^{1-\rho} - \delta K_{t-1}$$

Technology shock

TFP: Total Factor Productivity

$$\log Z_t = (1-\psi)\log \bar{Z} + \psi \log Z_{t-1} + \varepsilon_t \qquad \varepsilon_t \sim iid\, N(0, \sigma^2)$$

TFP: Total Factor Productivity

$$\log Z_t = (1-\psi)\log \bar{Z} + \psi \log Z_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim i.i.d.N(0, \sigma^2)$$

$$Z_{t-1} = Z_t = E_t Z_{t+1} = \bar{Z}, \quad \bar{\varepsilon} = E(\varepsilon_t) = 0$$

$$0 < \psi < 1$$

$$\log \bar{Z} = 0 \Rightarrow \bar{Z} = 1$$

$$\max_{C_t, N_t, K_t} \sum_{t=0}^{\infty} E_t \beta^t \left[\frac{C_t^{1-\eta} - 1}{1-\eta} \right]$$

s.t.

$$C_t + K_t = Z_t K_{t-1}^{\rho} N_t^{1-\rho} + (1-\delta) K_{t-1}, \quad N_t = 1$$

$$\log Z_t = (1-\psi)\log \bar{Z} + \psi \log Z_{t-1} + \varepsilon_t$$

$$K_{-1}, Z_0$$

$$\varepsilon_t \sim i.i.d.N(0, \sigma^2)$$

$$\mathcal{L} = E_t \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\eta} - 1}{1-\eta} + \lambda_t \left[Z_t K_{t-1}^{\rho} + (1-\delta) K_{t-1} - C_t - K_t \right] \right\}$$

$$\frac{\partial \mathcal{L}}{\partial C_t} = \cancel{\beta^t} \left\{ C_t^{-\eta} - \lambda_t \right\} = 0 \qquad (1)\ \lambda_t = C_t^{-\eta}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_t} = \cancel{\beta^t} \left\{ Z_t K_{t-1}^{\rho} + (1-\delta) K_{t-1} - C_t - K_t \right\} = 0 \qquad (2)$$

$$\frac{\partial \mathcal{L}}{\partial K_t} = -\cancel{\beta^t} \lambda_t + \cancel{\beta^{t+1}}^{\,\beta} E_t \left[\lambda_{t+1} (\rho Z_{t+1} K_t^{\rho-1} + (1-\delta)) \right] = 0 \qquad (3)$$

(1), (3)

$$C_t^{-\eta} = \beta E_t C_{t+1}^{-\eta} \left[\rho Z_{t+1} K_t^{\rho-1} + (1-\delta) \right]$$

(1) $$\frac{C_t^{-\eta}}{\lambda_t} = 1$$

$$C_t^{-\eta} = \beta E_t C_{t+1}^{-\eta} \left[\rho Z_{t+1} K_t^{\rho-1} + (1-\delta) \right]$$

$$\frac{C_t^{-\eta}}{\beta E_t C_{t+1}^{-\eta}} = E_t \left[\underbrace{\rho Z_{t+1} K_t^{\rho-1}}_{r_t} + (1-\delta) \right]$$

$$\frac{C_t^{-\eta}}{\beta E_t C_{t+1}^{-\eta}} = \left[\overbrace{r_t + 1 - \delta}^{R_t} \right]$$

$$\frac{C_t^{-\eta}}{\beta E_t C_{t+1}^{-\eta}} = \overbrace{\left[r_t + 1 - \delta \right]}$$

$$r_t = E_t\left[\rho z_{t+1} k_t^{\rho-1}\right] + 1 - \delta$$

$$\left(\frac{C_t}{E_t C_{t+1}}\right)^{-\eta} = \beta R_t$$

$$\frac{C_t}{E_t C_{t+1}} = \left(\beta R_t\right)^{-\frac{1}{\eta}}$$

Epstein – Zin preference

$$\lim_{T \to \infty} E_0\left[\beta^T C_T^{-\eta} K_T\right] = 0$$

Transversality condition

(1) $$1 = E_t\left[\beta\left(\frac{C_t}{C_{t+1}}\right)^{\eta} R_{t+1}\right]$$

(2) $$C_t = Z_t K_{t-1}^{\rho} + (1-\delta) K_{t-1} - K_t$$

(3) $$R_t = \rho Z_t K_{t-1}^{\rho-1} + (1-\delta)$$

(4) $$\log Z_t = (1-\psi)\log \bar{Z} + \psi \cdot \log Z_{t-1} + \varepsilon_t$$

$$X_t = X_{t-1} = E_t X_{t+1} = \bar{X}$$

Steady State:

(1) $$1 = \beta\left(\cancel{\frac{\bar{C}}{\bar{C}}}^{\,1}\right)^{\eta} \bar{R} \Rightarrow \bar{R} = \frac{1}{\beta}$$

(2) $$\bar{C} = \bar{Z} \bar{K}^{\rho} + (1-\delta)\bar{K} - \bar{K}$$

(3) $$\bar{R} = \rho \bar{Z} \bar{K}^{\rho-1} + (1-\delta)$$

(4) $$\bar{Z} = 1$$

معادلات را از معلوم ترین به مجهول ترین بازنویسی می کنید.

(1) $$\bar{Z} = 1$$

$$\beta = \frac{1}{1+\rho} = \frac{1}{1+\bar{r}}$$

$$\bar{r} = \rho$$

② $$\bar{R} = \frac{1}{\beta}$$

③ $$\bar{K} = \left(\frac{\rho \bar{Z}}{\bar{R}-1+\delta}\right)^{\frac{1}{1-\rho}}$$

④ $$\bar{Y} = \bar{Z}\bar{K}^{\rho}$$

⑤ $$\bar{C} = \bar{Y} - \delta \bar{K} \Rightarrow \bar{Y} = \bar{C} + \delta \bar{K}$$

The competitive Equilibrium:

مصرف کننده:

$$\max_{C_t, K_t} E\left[\sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\eta} - 1}{1-\eta}\right]$$

s.t.

$$C_t + I_t = W_t N_t^s + r_t K_{t-1}^s$$

$$N_t^s = 1$$

$$K_t^s = (1-\delta) K_{t-1}^s + I_t$$

تولیدکننده:

$$\max_{N_t, K_{t-1}} \Pi_t = Z_t K_{t-1}^{d\,\rho} N_t^{d\,1-\rho} - W_t N_t^d - r_t K_{t-1}^d$$

شرط تسویه بازارها

$$N_t^s = N_t^d = N_t = 1$$ بازار کار

$$K_{t-1}^s = K_{t-1}^d = K_{t-1}$$ بازار سرمایه

بازار محصول

✓ $$C_t + K_t = Z_t K_{t-1}^{\rho} + (1-\delta) K_{t-1}$$

$$\begin{cases} Y_t = C_t + I_t \\ Y_t = Z_t K_{t-1}^{\rho} \\ K_t = (1-\delta) K_{t-1} + I_t \end{cases}$$

تولیدکننده

$$\frac{\partial \Pi_t}{\partial N_t} = (1-\rho) Z_t K_{t-1}^{\rho} N_t^{-\rho} - W_t = 0$$

$$\frac{\partial \Pi_t}{\partial K_{t-1}} = \rho Z_t K_{t-1}^{\rho-1} N_t^{1-\rho} - r_t = 0$$

$$W_t = MP_{N_t} = \frac{(1-\rho) Y_t}{N_t} = (1-\rho) \frac{Z_t K_{t-1}^{\rho} N_t^{1-\rho}}{N_t}$$

$$r_t = MP_{K_t} = \rho \frac{Y_t}{\cdot} = \rho \frac{Z_t K_{t-1}^{\rho} N_t^{1-\rho}}{}$$

$$w_t = MPN_t$$

$$r_t = MPK_t = \rho \frac{Y_t}{K_{t-1}} = \rho \frac{Z_t K_{t-1}^{\rho} N_t^{1-\rho}}{K_{t-1}}$$

مصرف شده

$$① \quad \frac{\partial \mathcal{L}}{\partial \lambda_t} = w_t + r_t K_{t-1} + (1-\delta) - C_t - K_t = 0$$

$$② \quad \frac{\partial \mathcal{L}}{\partial C_t} = C_t^{-\rho} - \lambda_t = 0$$

$$③ \quad \frac{\partial \mathcal{L}}{\partial K_t} = -\lambda_t + \beta E_t \lambda_{t+1} \underbrace{[r_t + (1-\delta)]}_{R_{t+1}} = 0$$

$$X_t = A X_{t-1} + \varepsilon_t$$

$$\dot{x}_t = \frac{X_t - X_{t-1}}{X_{t-1}} \qquad \text{غیرخطی}$$

$$\dot{x}_t \simeq \ln X_t - \ln X_{t-1} = \ln \frac{X_t}{X_{t-1}} \qquad \text{نرخ رشد ناخالص}$$

لگاریتم - خطی کردن مدل حول وضعیت پایدار:

$$X_t = X_{t-1} = E_t X_{t+1} = \bar{X}$$

$$① \quad \hat{x}_t = \ln X_t - \ln \bar{X}$$ درصد انحرافات X_t از وضعیت پایدار

$$\hat{x}_t = \ln \frac{X_t}{X_{t-1}}$$

$$\hat{x}_t = \frac{X_t - \bar{X}}{\bar{X}} = \frac{X_t}{\bar{X}} - 1 = \hat{x}_t$$

$$\frac{X_t}{\bar{X}} = 1 + \hat{x}_t$$

$$② \quad \left.\begin{aligned} X_t &= \bar{X}(1+\hat{x}_t) \\ Y_t &= \bar{Y}(1+\hat{y}_t) \end{aligned}\right\} \quad \hat{x}_t \hat{y}_t \simeq 0$$

$$① \quad \hat{x}_t = \ln \frac{X_t}{\bar{X}} \Rightarrow \frac{X_t}{\bar{X}} = e^{\hat{x}_t}$$

$$③ \quad X_t = \bar{X} e^{\hat{x}_t}$$

$$Y_t = Z_t K_{t-1}^{\alpha} N_t^{1-\alpha}$$
$$\bar{Y} = \bar{Z} \bar{K}^{\alpha} \bar{N}^{1-\alpha}$$

$$\ln Y_t = \ln Z_t + \alpha \ln K_{t-1} + (1-\alpha) \ln N_t$$
$$\ln \bar{Y} = \ln \bar{Z} + \alpha \ln \bar{K} + (1-\alpha) \ln \bar{N}$$

$$\ln Y_t - \ln \bar{Y} = \ln Z_t - \ln \bar{Z} + \alpha(\ln K_t - \ln \bar{K}) + (1-\alpha)$$

$$\ln \bar{Y} = \ln \bar{Z} + \alpha \ln \bar{K} + (1-\alpha) \ln \bar{N}$$

$$\ln Y_t - \ln \bar{Y} = \ln Z_t - \ln \bar{Z} + \alpha(\ln K_{t-1} - \ln \bar{K}) + (1-\alpha)(\ln N_t - \ln \bar{N})$$

$$\hat{y}_t = \hat{z}_t + \alpha \hat{k}_{t-1} + (1-\alpha)\hat{n}_t$$

$$Y_t = C_t + I_t$$

$$\bar{Y} = \bar{C} + \bar{I} \Rightarrow \frac{\bar{C}}{\bar{Y}} + \frac{\bar{I}}{\bar{Y}} = 1$$

$$\bar{Y}(1+\hat{y}_t) = \bar{C}(1+\hat{c}_t) + \bar{I}(1+\hat{i}_t)$$

$$\cancel{\bar{Y}} + \bar{Y}\hat{y}_t = \cancel{\bar{C}} + \bar{C}\hat{c}_t + \cancel{\bar{I}} + \bar{I}\hat{i}_t$$

$$\hat{y}_t = \frac{\bar{C}}{\bar{Y}}\hat{c}_t + \frac{\bar{I}}{\bar{Y}}\hat{i}_t$$

$$Y_t = a + X_t \quad , \quad \bar{Y} = a + \bar{X}$$

$$\bar{Y}(1+\hat{y}_t) = a + \bar{X}(1+\hat{x}_t)$$

$$\cancel{\bar{Y}} + \bar{Y}\hat{y}_t = \cancel{a} + \cancel{\bar{X}} + \bar{X}\hat{x}_t$$

$$\bar{Y}\hat{y}_t = \bar{X}\hat{x}_t$$

$$\hat{y}_t = \frac{\bar{X}}{\bar{Y}}\hat{x}_t$$

$$X_t + a = (1-b)\frac{Y_t}{Z_t}$$

$$\bar{X} + a = (1-b)\frac{\bar{Y}}{\bar{Z}}$$

$$\ln(X_t + a) = \ln(1-b) + \ln Y_t - \ln Z_t$$

$$\widehat{(X_t + a)} = \hat{y}_t - \hat{z}_t$$

$$\widehat{X_t + a} = \frac{X_t + a - (\bar{X} + a)}{\bar{X} + a} = \frac{X_t - \bar{X}}{\bar{X} + a} \times \frac{\bar{X}}{\bar{X}}$$

$$\widehat{X_t + a} = \frac{\bar{X}}{\bar{X} + a} \underbrace{\left(\frac{X_t - \bar{X}}{\bar{X}}\right)}_{\hat{x}_t} = \frac{\bar{X}}{\bar{X} + a}\hat{x}_t = \hat{y}_t - \hat{z}_t$$

(1) $$1 = \beta E_t \left(\frac{C_{t+1}}{C_t}\right)^{-\eta} R_{t+1}$$

$$1 = \beta \left(\frac{\bar{C}}{\bar{C}}\right)^{-\eta} \bar{R}$$

$$0 = \eta [c_t - E_t c_{t+1}] + \hat{r}_{t+1}$$

② $$c_t = \underbrace{z_t k_{t-1}^{\rho}}_{Y_t} + (1-\delta) k_{t-1} - k_t$$

$$Y_t = z_t k_{t-1}^{\rho} \qquad \hat{k}_{t-1} = \ln K_{t-1} - \ln \bar{K}$$

$$\hat{y}_t = \hat{z}_t + \rho \hat{k}_{t-1}$$

$$\bar{c} = \bar{Y} + (1-\delta)\bar{k} - \bar{k}$$

$$\bar{c}(\cancel{1} + \hat{c}_t) = \bar{Y}(\cancel{1} + \hat{y}_t) + (1-\delta)\bar{k}(\cancel{1} + \hat{k}_{t-1}) - \bar{k}(\cancel{1} + \hat{k}_t)$$

$$\hat{c}_t = \frac{\bar{Y}}{\bar{c}} \hat{y}_t + \frac{(1-\delta)\bar{k}}{\bar{c}} \hat{k}_{t-1} - \frac{\bar{k}}{\bar{c}} \hat{k}_t$$

$$\hat{c}_t = \frac{\bar{Y}}{\bar{c}} (\hat{z}_t + \rho \hat{k}_{t-1}) + \frac{(1-\delta)\bar{k}}{\bar{c}} \hat{k}_{t-1} - \frac{\bar{k}}{\bar{c}} \hat{k}_t$$

③ $$R_t = \rho z_t k_{t-1}^{\rho-1} + (1-\delta)$$

$$\bar{R}(\cancel{1} + \hat{r}_t) = \rho \bar{z} \bar{k}^{\rho-1}(\cancel{1} + \hat{z}_t + (\rho-1)\hat{k}_{t-1}) \cancel{+(1-\delta)}$$

$$\bar{R}\hat{r}_t = \rho \bar{z} \bar{k}^{\rho-1}(\hat{z}_t + (\rho-1)\hat{k}_{t-1})$$

$$\hat{r}_t = \frac{\rho \bar{z} \bar{k}^{\rho-1}}{\bar{R}} (\hat{z}_t + (\rho-1) \hat{k}_{t-1})$$

$$\hat{r}_t = (1-\beta(1-\delta))(\hat{z}_t - (1-\rho)\hat{k}_{t-1})$$

④ $$\hat{z}_t = \psi \hat{z}_{t-1} + \varepsilon_t$$

① $$0 = E_t[\eta(\hat{c}_t - \hat{c}_{t+1}) + \hat{r}_{t+1}]$$

② $$\hat{c}_t = \frac{\bar{Y}}{\bar{c}} \hat{z}_t + \frac{\bar{k}}{\beta \bar{c}} \hat{k}_{t-1} - \frac{\bar{k}}{\bar{c}} \hat{k}_t$$

③ $$\hat{r}_t = (1-\beta(1-\delta))(\hat{z}_t - (1-\rho)\hat{k}_{t-1})$$

④ $$\hat{z}_t = \psi \hat{z}_{t-1} + \varepsilon_t$$

حل سیستم معادلات با روش ضرایب نامعین:

$$\hat{k}_t = \nu_{kk} \hat{k}_{t-1} + \nu_{kz} \hat{z}_t$$

$$\hat{r}_t = \nu_{rk} \hat{k}_{t-1} + \nu_{rz} \hat{z}_t$$

$$\hat{c}_t = \nu_{ck} \hat{k}_{t-1} + \nu_{cz} \hat{z}_t$$

$$E\,\hat{z}_{t+1} = \psi\,\hat{z}_t$$

(2) $$\hat{c}_t = \left(1+\delta\frac{\bar{k}}{\bar{c}}\right)\hat{z}_t + \frac{\bar{k}}{\beta\bar{c}}\hat{k}_{t-1} - \frac{\bar{k}}{\bar{c}}\hat{k}_t$$

$$\hat{c}_t = v_{ck}\hat{k}_{t-1} + v_{cz}\hat{z}_t = \left(1+\delta\frac{\bar{k}}{\bar{c}}\right)\hat{z}_t + \frac{\bar{k}}{\beta\bar{c}}\hat{k}_{t-1} - \frac{\bar{k}}{\bar{c}}\left(v_{kk}\hat{k}_{t-1} + v_{kz}\hat{z}_t\right)$$

$$\boxed{v_{cz} = \frac{\bar{y}}{\bar{c}} - \frac{\bar{k}}{\bar{c}}v_{kz}}$$

$$\boxed{v_{ck} = \left(\frac{1}{\beta} - v_{kk}\right)v_{kz}}$$

(3) $$\hat{r}_t = (1-\beta(1-\delta))\left(\hat{z}_t - (1-\rho)\hat{k}_{t-1}\right) = v_{rk}\hat{k}_{t-1} + v_{rz}\hat{z}_t$$

$$\boxed{v_{rk} = -(1-\rho)(1-\beta(1-\delta))}$$

$$\boxed{v_{rz} = 1-\beta(1-\delta)}$$

(1) $$0 = E_t\left[\eta(\hat{c}_t - \hat{c}_{t+1}) + \hat{r}_{t+1}\right]$$

$$0 = E_t\Big[\eta\left(v_{ck}\hat{k}_{t-1} + v_{cz}\hat{z}_t\right) - \left(v_{ck}\hat{k}_t + v_{cz}\hat{z}_{t+1}\right) + \left(v_{rk}\hat{k}_t + v_{rz}\hat{z}_{t+1}\right)\Big]$$

$$= (v_{rk} - \eta v_{ck})\hat{k}_t + \eta v_{ck}\hat{k}_{t-1} + \left((v_{rz} - \eta v_{cz})\psi + \eta v_{cz}\right)\hat{z}_t$$

$$0 = \left((v_{rk} - \eta v_{ck})v_{kk} + \eta v_{ck}\right)\hat{k}_{t-1} + \left((v_{rk} - \eta v_{ck})v_{kz} + (v_{rz} - \eta v_{cz})\psi + \eta v_{cz}\right)\hat{z}_t$$

$$\boxed{(v_{rk} - \eta v_{ck})v_{kk} + \eta v_{ck} = 0}$$

$$\boxed{((v_{rk} - \eta v_{ck})v_{kz} + (v_{rz} - \eta v_{cz})\psi + \eta v_{cz} = 0}$$

$$0 = \left(-(1-\beta(1-\delta))(1-\rho) - \eta\left(\frac{1}{\beta} - v_{kk}\right)\frac{\bar{k}}{\bar{c}}\right)v_{kk} + \eta\left(\frac{1}{\beta} - v_{kk}\right)\frac{\bar{k}}{\bar{c}}$$

$$0 = v_{kk}^2 - \gamma v_{kk} + \frac{1}{\beta}$$

$$\gamma = (1-\beta(1-\delta))(1-\rho)\frac{\bar{c}}{\rho\bar{k}} + 1 + \frac{1}{\beta}$$

$$v_{kk} = \frac{\gamma}{2} - \sqrt{\left(\frac{\gamma}{2}\right)^2 - \frac{1}{\beta}}$$

$$y_t = \alpha + \rho\, y_{t-1} + \varepsilon_t$$

$$\hat{k}_t = \nu_{kk}\hat{k}_{t-1} + \nu_{kz}\hat{z}_t$$

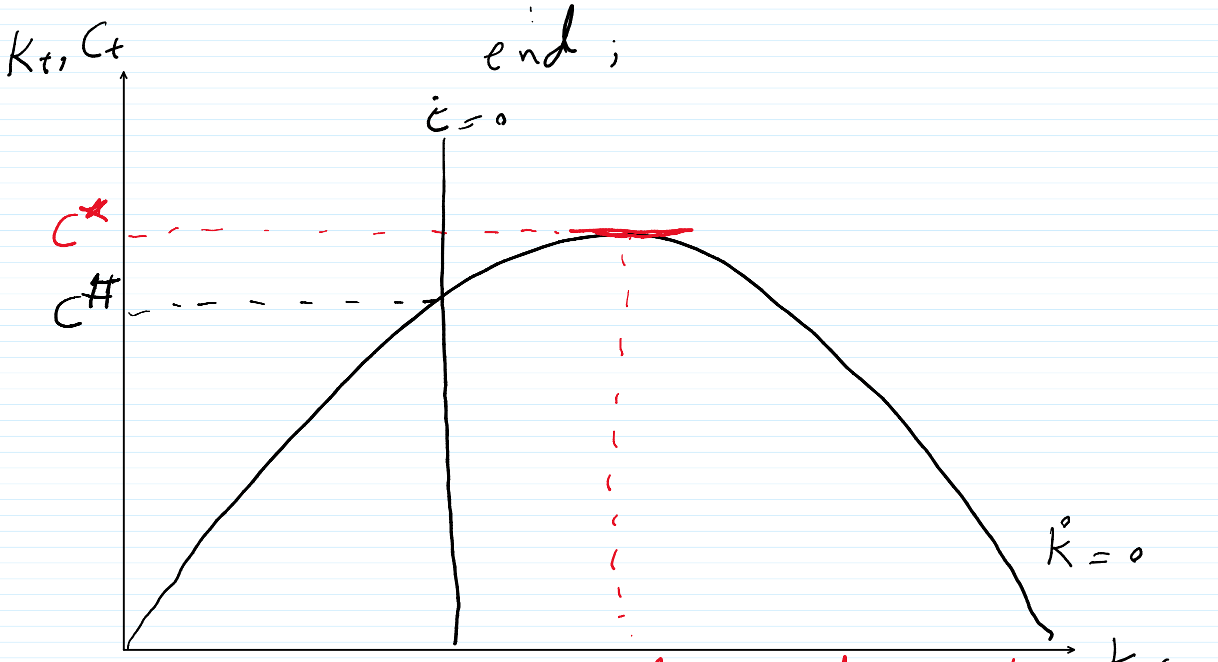
$$C_t - hC_{t-1} \qquad \text{ایجاد:}$$

Dynare:

بلوک مقدمه

Var	متغیر درون‌زا
Varexo	// برون‌زا
Parameters	پارامتر

```
model(linear);
:
end;
```



$$K^\# \qquad K^* = \text{Golden Rule}$$

$$\frac{1}{1+\rho} = \beta = \frac{1}{R} = \frac{1}{1+\bar{r}}$$

$$\bar{r} = \rho \longrightarrow \text{Deep Parameter}$$

$$\rho > 0 \Rightarrow \bar{r} > 0$$

$$\rho > 0 \Rightarrow \bar{r} > 0$$

$$\rho = 0 \Rightarrow \bar{r} = 0$$

Exogenous Nominal Rigidity:

Fixed Prices:

$$Y = F(L), \quad F'(\cdot) > 0, \quad F''(\cdot) \leq 0$$

$$Y = C$$

$$U = \sum_{t=0}^{\infty} \beta^t \left[U(C_t) + \Gamma\left(\frac{M_t}{P_t}\right) - V(L_t) \right]$$

پول

سطح قیمت

Real Money Balances: مانده حقیقی پول

$$0 < \beta < 1$$

Search & Match

$$U(C_t) = \frac{C_t^{1-\theta}}{1-\theta}, \quad \theta > 0$$

$$\Gamma\left(\frac{M_t}{P_t}\right) = \frac{(M_t/P_t)^{1-\chi}}{1-\chi}, \quad \chi > 0$$

$$+ B_t + M_t = W_t L_t + (1 + i_{t-1}) B_{t-1} + M_{t-1}$$

$$\frac{B_t}{P_t} + \frac{M_t}{P_t} = \frac{W_t}{P_t} L_t + (1 + i_{t-1}) \frac{B_{t-1}}{P_t} + \frac{M_{t-1}}{P_t}$$

$$U = \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\theta}}{1-\theta} + \frac{(M_t/P_t)^{1-\chi}}{1-\chi} - V(L_t) \right]$$

$$U = \sum_{t=0} \beta^t \left[\frac{\quad}{1-\theta} + \frac{\quad}{1-\chi} - v(L_t) \right]$$

$$b_t + m_t = w_t L_t + (1+i_{t-1}) \frac{b_{t-1}}{1+\pi_t} + \frac{m_{t-1}}{1+\pi_t}, \quad w_t = \frac{W_t}{P_t}$$

$$b_t = \frac{B_t}{P_t}, \quad \frac{B_{t-1}}{P_t} = \underbrace{\frac{B_{t-1}}{P_{t-1}}}_{b_{t-1}} \frac{P_{t-1}}{P_t} = \frac{b_{t-1}}{P_t / P_{t-1}} = \frac{b_{t-1}}{1+\pi_t}$$

$$\pi_t = \frac{P_t - P_{t-1}}{P_{t-1}} = \frac{P_t}{P_{t-1}} - 1 \Rightarrow \frac{P_t}{P_{t-1}} = 1 + \pi_t$$

$$\sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\theta}}{1-\theta} + \frac{m_t^{1-\chi}}{1-\chi} - v(L_t) + \lambda_t \left(w_t L_t + \frac{m_{t-1}}{1+\pi_t} + \frac{(1+i_{t-1})}{1+\pi_t} b_{t-1} - c_t - b_t - m_t \right) \right]$$

$$= c_t^{-\theta} - \lambda_t = 0$$

$$= m_t^{-\chi} - \lambda_t + \beta E_t \frac{\lambda_{t+1}}{1+\pi_{t+1}} = 0$$

$$= v'(L) - w_t \lambda_t = 0$$

$$= -\lambda_t + \beta E_t \frac{(1+i_t) \lambda_{t+1}}{1+\pi_{t+1}} = 0$$

Max

s.t.

$C_t + b$

$b_t =$

$m_t =$

$\mathcal{L} =$

$+ \frac{(1+}{}$

① $\frac{\partial \mathcal{L}}{\partial C_t}$

② $\frac{\partial \mathcal{L}}{\partial m_t}$

③ $\frac{\partial \mathcal{L}}{\partial L_t}$

④ $\frac{\partial \mathcal{L}}{\partial b_t}$

$1 + r_{t+1}$

$\ldots = \ldots = 0$

$$\frac{V'(L)}{C_t^{-\theta}} = w_t$$

عرضه کار

$$\beta E_t \frac{\lambda_{t+1}}{1+\pi_{t+1}} = \frac{\lambda_t}{1+i_t}$$

$$m_t^{-\chi} = \lambda_t - \frac{\lambda_t}{1+i_t} = \frac{i_t}{1+i_t} C_t^{-\theta}$$

$$\frac{m_t^{-\chi}}{C_t^{-\theta}} = \frac{i_t}{1+i_t} \Rightarrow m_t^{-\chi} = \frac{i_t}{1+i_t} C_t^{-\theta}$$

تقاضای پول

$$C_t^{-\theta} = \beta E_t \underbrace{\frac{(1+i_t)}{1+\pi_{t+1}}}_{1+r_{t+1}} C_{t+1}^{-\theta} \Rightarrow$$

معادله اویلر

که نرخ اجاره سرمایه

$$\ldots + r_{t+1} = E_t \frac{1+i_t}{1+\pi_{t+1}}$$ Fisher Eq.

$$\ldots E_t r_{t+1})(1+E_t \pi_{t+1}) = 1 + i_t$$

$$\ldots E_t r_{t+1} + E_t \pi_{t+1} + E_t r_{t+1} E_t \pi_{t+1} = \cancel{1} + i_t$$

$$\ldots r_{t+1} = i_t - E_t \pi_{t+1} + E_t r_{t+1} E_t \pi_{t+1}$$

∂b_t

⑤ $\frac{\partial \mathcal{L}}{\partial \lambda_t}$

①, ③

④

①, ②, ④

①, ④

(

$(1+$

$\cancel{1}+$

E_t

$$\cancel{Y = C}$$
$$\cancel{Y = C + S}$$
$$\cancel{S = 0}$$

$$Y_t = C_t$$

$$1 + r_t$$

$$Y_t^{-\theta} = \beta \, E_t \frac{(1+i_t)}{1+\pi_{t+1}} Y_{t+1}^{-\theta}$$

New Keynesian IS

$$m_t^{-\alpha} = \frac{i_t}{1+i_t} Y_t^{-\theta}$$

$$m_t = \frac{M_t}{P_t}$$ پایه پولی

$$m_t^s = \frac{M^s_t}{P_t}$$ عرضه پول

$$\bar{m}_t^s = m_t^d = \bar{m}_t$$

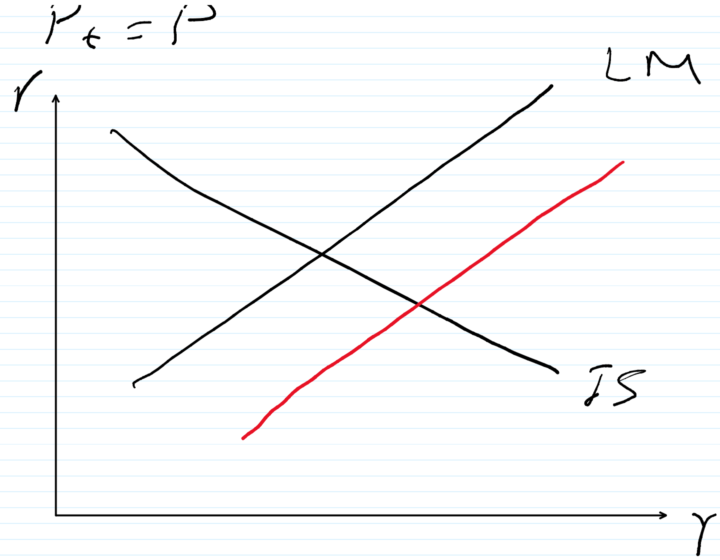
$$\bar{m}_t^{-\alpha} = \left(\frac{i_t}{1+i_t}\right) Y_t^{-\theta} \qquad LM$$

$$\Longrightarrow \ln Y_t = a + \ln E_t Y_{t+1} - \frac{1}{\theta} r \qquad IS$$

$$a = -\left(\frac{1}{\theta}\right) \ln \beta$$

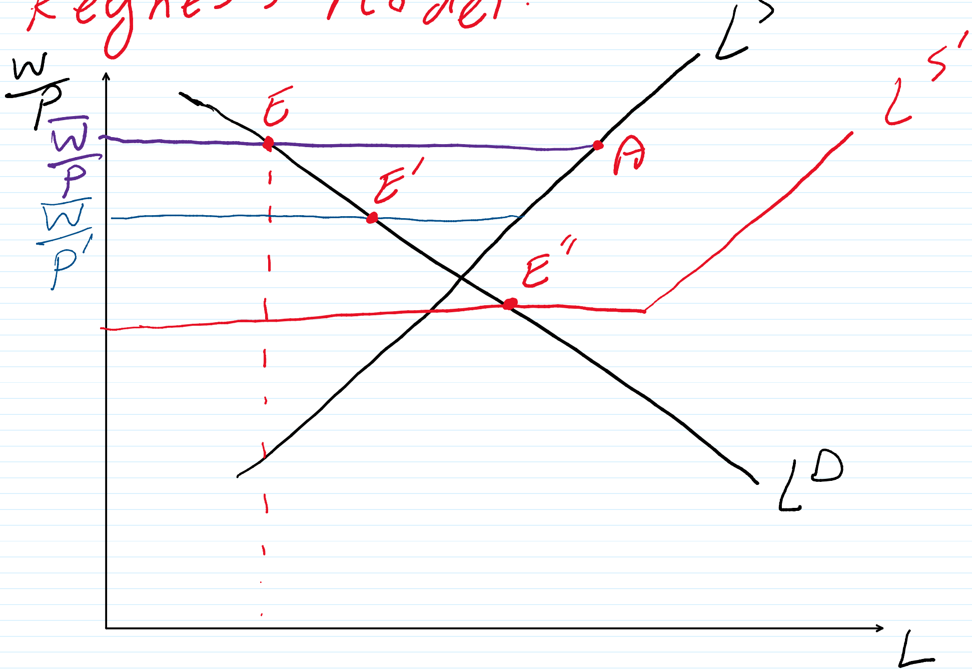
$$P_t = \bar{P}$$

r. , LM



$M \uparrow$

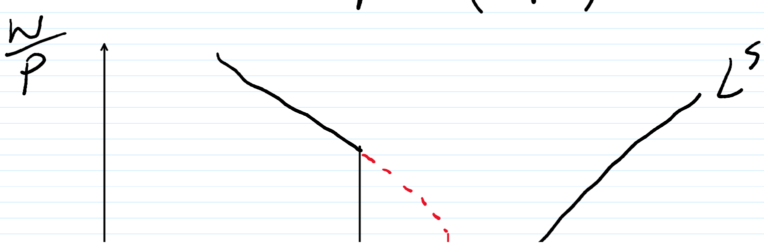
2: Keynes's model:



$$L = L^s\left(\frac{W}{P}\right) \quad L^{s\prime}(\cdot) > 0$$

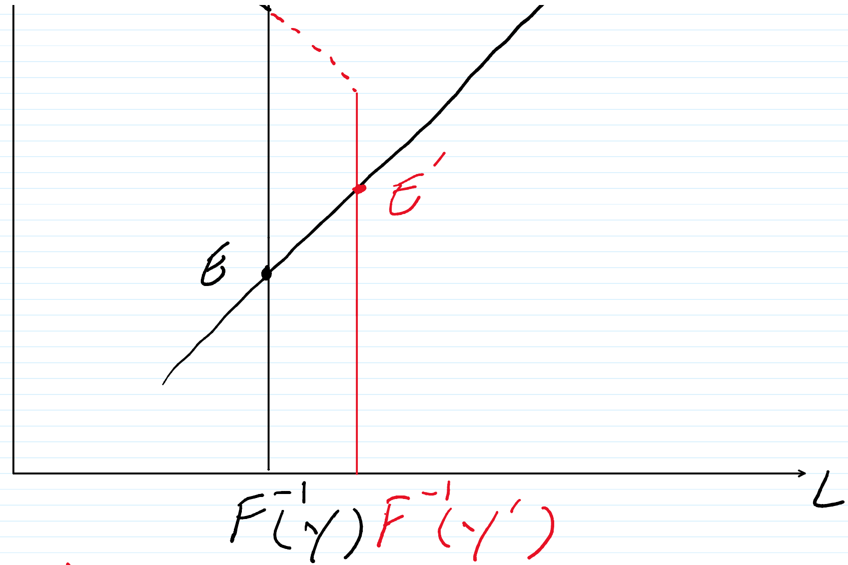
$$Y = F(L)$$

$$L = F^{-1}(Y)$$



Case 2

$p' > p$



Sticky prices, flexible wages and Real

r Market imperfections:

$$\underbrace{\pi_t - E_t \pi_{t+1}}_{\text{شکاف تورمی}} = f(u_t - \underbrace{\bar{u}}_{\text{بیکاری طبیعی}})$$

NAIRU

on-Accelarating Inflation Rate of Unemployment

$$\pi_t - E_t \pi_{t+1} = g(\underbrace{Y_t - Y^P}_{\text{شکاف تولید}})$$

cy wage

ge Function

Case 3:

Labo

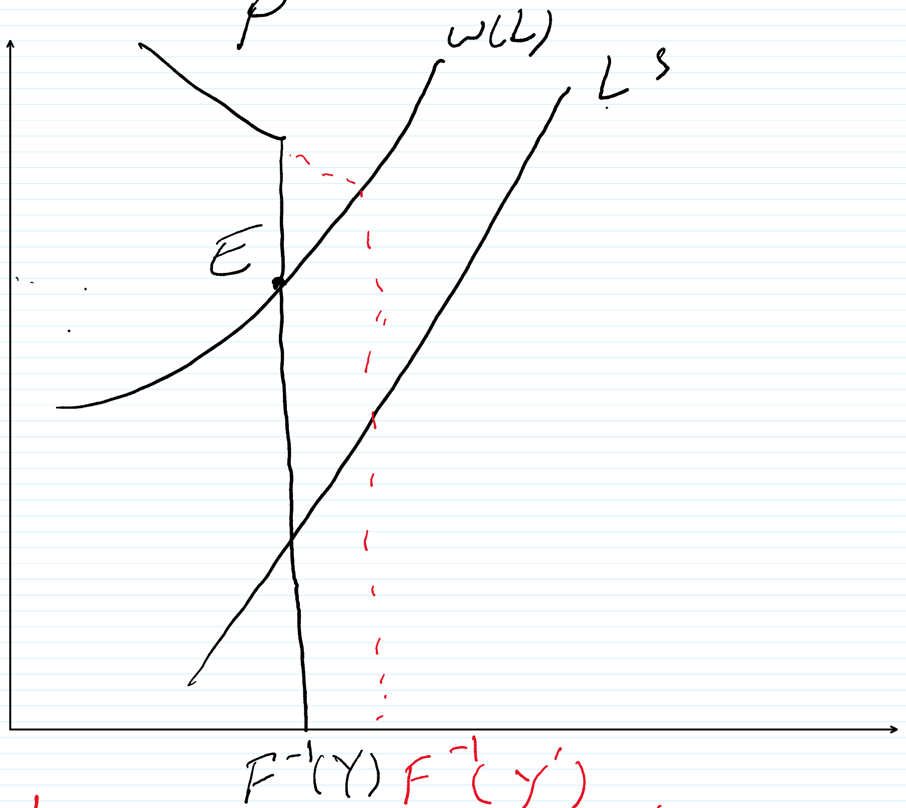
N

Efficien

real_wa

ge

$$\frac{W}{P} = \omega(L) \qquad \omega'(\cdot) > 0$$



icky Wage, Flexible Prices, and

Competition:

$$= P = MC$$

$$R = P(1+\eta)$$

$$= \frac{1}{1+\eta} MC$$

$$= \frac{1}{\quad} > 1 \qquad \text{Markup}$$

real-wa

$\frac{w}{P}$

Case 4: St

Imperfect

$MR =$

$MC = M$

$P =$

$\eta < 0$

$$= \frac{1}{1+\eta} > 1 \quad \text{Markup}$$

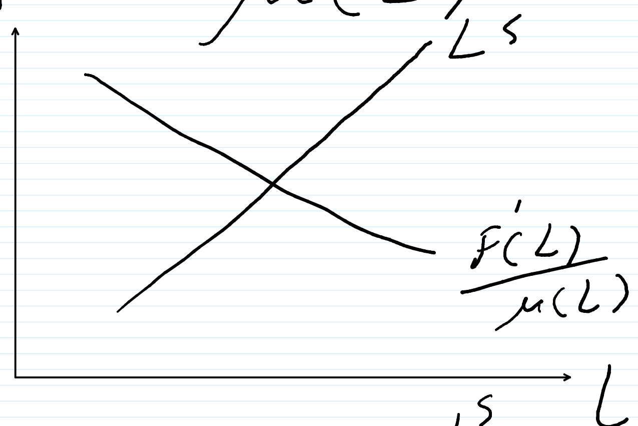
$$\frac{\partial Y}{\partial L} = W$$

$$MP_L = W$$

$$= \frac{W}{\frac{\partial Y}{\partial L}} = MC = \frac{W}{\underbrace{MP_L}_{F'(L)}}$$

$$P = \mu(L) \frac{W}{F'(L)}$$

$$\frac{W}{P} = \frac{F'(L)}{\mu(L)}$$



μ

$P \frac{\partial}{\partial}$

$P\ M$

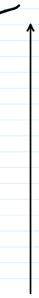
P

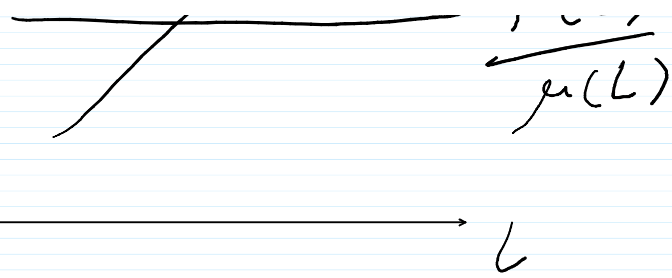
P

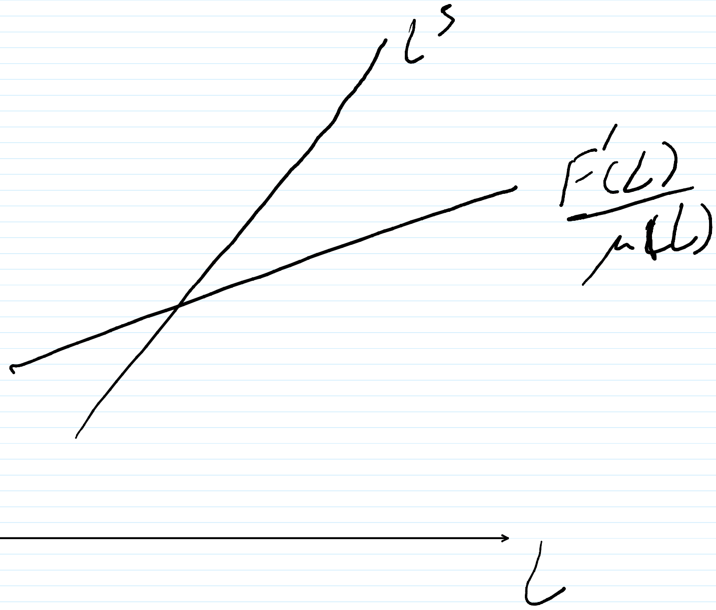
w

$\frac{w}{P}$

$\frac{w}{P}$







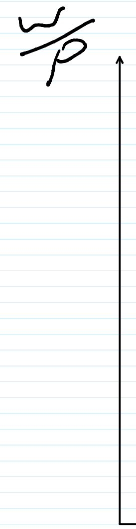
$$E_t \pi_{t+1} = f(Y_t - \bar{Y})$$

-1

$$\pi_t - \pi_{t-1} = f(\underbrace{Y_t - \bar{Y}}_{y_t}) = \lambda y_t \quad \lambda > 0 \quad \text{NKPC}$$

$$E_t \pi_{t+1} + \frac{1}{1+\phi}\pi_{t-1} + \lambda y_t$$

$$r_t = b\, y_t \qquad b > 0 \qquad \text{Taylor Rule}$$



$\pi_t -$

$\pi_t -$

π

$\pi_t = \frac{\phi}{1+\phi}$

$$y_t = E_t y_{t+1} - \frac{1}{\theta} r_t + u_t^{IS} \qquad \theta > 0$$

$$u_t^{IS} = \rho_{IS} u_{t-1}^{IS} + e_t^{IS} \qquad -1 < \rho_{IS} < 1$$

$$r_t = r_{t-1} + \lambda y_t$$

$$- \frac{1}{\theta} b y_t + u_t^{IS}$$

$$E_t y_{t+1} + u_t$$

$$\underbrace{\frac{\theta}{\theta + b}}_{\phi} E_t y_{t+1} + \underbrace{\frac{\theta}{\theta + b}}_{\phi} u_t^{IS}$$

$$E_t y_{t+1} + \phi u_t^{IS}$$

$$\frac{\phi}{1 - \phi \rho_{IS}} u_t^{IS} + \lim_{s \to \infty} \phi^s E_t y_{t+s+}$$

$$y$$

$$u$$

$$r$$

$$y_t = E_t y_{t+1}$$

$$\left(\frac{\theta + b}{\theta}\right) y_t =$$

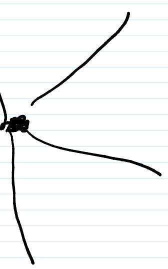
$$y_t = \frac{\theta}{\theta}$$

$$y_t = \phi$$

$$y_t = \frac{\phi}{1}$$

$$\frac{\quad}{\ldots b - \theta f^{IS}} \; u_t^{IS}$$

$$\ldots_{t-1} + \frac{\lambda \theta}{\theta + b - \theta f^{IS}} \; u_t^{IS}$$



$$= E_t y_{t+1} - \frac{1}{\theta}\left(i_t - E_t \pi_{t+1} \right) + u_t^{IS}$$

$$= b_y y_t + b_\pi \pi_t + u_t^{i}$$

$$y_1 = \frac{\theta}{\theta +}$$

$$\pi_t = \pi_t$$

$y_t =$

$i_t =$